End-of-Year (EoY) FRQ practice 2021/05/07 21h37 EDT

USE AT YOUR OWN RISK. DO NOT ASSUME EVERY PROBLEM STATEMENT IS DEBUGGED.

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1 AP PHYSICS 1

1.1 Kinematics

1.1.1 **PLR**. (Constant velocity model), key online.

Let the $+x$ direction point along a straight racetrack from the start line to the finish line. At the start of a race, Car A and Car B are stopped at the start line. Each car almost immediately gets up to speed and then continues to move at constant velocity through the end of the race, which happens when Car A crosses the finish line first.

Indicate whether Car A's average x-velocity during the race is greater than, less than, or the same as Car B's average x-velocity during the race.

___ Greater than ___ Less than ___ Same as

Explain your reasoning.

(video key is at http://davidliao.com).

1.1.2 **PLR**. Constant acceleration model (practice not merely narrating algebra)
A car is moving in the $+x$ direction along a straight highway at speed v. At time $t = 0$, the driver applies the brakes so that the car slows down with constant acceleration until the car comes to rest. Let time $t_{\frac{1}{2}}$ denote the time at which the car's speed has decreased to $\frac{1}{2}$ its original value, and let t_{STOP} denote the time at which the car is just then coming to rest. What is the ratio of the car's x-displacement from time $t_{\frac{1}{2}}$ to time t_{STOP} to the car's x-displacement from time $t = 0$ to time $t_{\frac{1}{2}}$? Explain your reasoning in a way that does not look merely like a block of algebra or like a block of algebra superficially disguised by English narration.

1.1.3 **QQT.** Toy car.
(12 points, suggested time 25 minutes) (inspired by 2019 AP Physics 1 FRQ 2) A toy car is programmed to travel in the $+x$ direction in two phases of motion. In the first phase, the car travels with constant velocity v_1 for a time duration Δt_1. In the second phase, the toy car continues to travel, but now with constant non-zero acceleration. At the end of the second phase of motion, which lasts for a time duration Δt_2, the toy car reaches a final velocity v_2 that is greater than v_1. In a set of experimental trials, time durations Δt_1 and Δt_2 are varied while total time $\Delta t_{\mathrm{TOT}} = \Delta t_1 + \Delta t_2$ and velocities v_1 and v_2 are held constant.

(a)

i. Suppose the duration Δt_1 is much greater than the duration Δt_2. Estimate the x-displacement through which the car travels during the experiment. Express your answer in terms of v_1, v_2, Δt_{TOT}, and fundamental constants, as appropriate.

Briefly explain your reasoning without deriving or manipulating equations.

ii. Now suppose the duration Δt_1 is much less than the duration Δt_2. Estimate the x-displacement through which the car travels during the experiment. Express your answer in terms of v_1, v_2, Δt_{TOT}, and fundamental constants, as appropriate.

Briefly explain your reasoning without deriving or manipulating equations.

(b) Now suppose neither time duration Δt_1 nor time duration Δt_2 is much greater than the other, but that they are not necessarily equal. On the axes below, draw a velocity vs. time graph for the car. Label axes and the values of times and velocities at the beginning and end of each phase of motion.

(c) Derive an equation for the x-displacement through which the car travels during the experiment in terms of v_1, v_2, Δt_1, Δt_{TOT}, and fundamental constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

(d) Consider the scenario from part (a)(ii), where the duration Δt_1 is much less than the duration Δt_2. Does your equation for the x-displacement of the car from part (c) agree with your reasoning in part (a)(ii)?

___ Yes   ___ No

Briefly explain your reasoning by addressing why, according to your equation, the x-displacement becomes (or approaches) a certain value when Δt_1 is much less than Δt_2.

(e) In one trial, the car travels through an overall displacement Δx, and the acceleration of the car during the second phase of motion is a_{I}. In a second trial, the car travels through the same overall displacement Δx through the same total time Δt_{TOT} and achieves the same final velocity v_2, but starts out with a slightly greater initial velocity v_1 (still less than v_2). The acceleration of the car during the second phase of motion during the second experiment is a_{II}. How do the two accelerations a_{I} and a_{II} compare to each other?

___ $a_{\text{I}} > a_{\text{II}}$ ___ $a_{\text{I}} = a_{\text{II}}$ ___ $a_{\text{I}} < a_{\text{II}}$

Briefly explain your reasoning. You may use (a) diagram(s) to support your explanation.

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1.1.3.1 Key

(12 points, suggested time 25 minutes) (inspired by 2019 AP Physics 1 FRQ 2)

A toy car is programmed to travel in the $+x$ direction in two phases of motion. In the first phase, the car travels with constant velocity v_1 for a time duration Δt_1. In the second phase, the toy car continues to travel, but now with constant non-zero acceleration. At the end of the second phase of motion, which lasts for a time duration Δt_2, the toy car reaches a final velocity v_2 that is greater than v_1. In a set of experimental trials, time durations Δt_1 and Δt_2 are varied while total time $\Delta t_{\mathrm{TOT}} = \Delta t_1 + \Delta t_2$ and velocities v_1 and v_2 are held constant.

(a)

i. *2 points*

Suppose the duration Δt_1 is much greater than the duration Δt_2. Estimate the x-displacement through which the car travels during the experiment. Express your answer in terms of v_1, v_2, Δt_{TOT}, and fundamental constants, as appropriate.

Briefly explain your reasoning without deriving or manipulating equations.

Correct answer: $v_1 \Delta t_{\mathrm{TOT}}$		
For a correct answer and attempt at a consistent justification		1 point
For correct reasoning		1 point
Example earning 1 point: $v_1 \Delta t_{\mathrm{TOT}}$. Because duration Δt_1 is much longer than duration Δt_2.		
Example earning 2 points: $v_1 \Delta t_{\mathrm{TOT}}$. Because Δt_1 is much greater than Δt_2, Δt_1 will last for the majority of the total duration Δt_{TOT}. Consequently, the velocity of the car will be v_1 for nearly all of Δt_{TOT}, so the x-displacement, which is the velocity times the duration, will be $v_1 \Delta t_{\mathrm{TOT}}$.		

ii. *2 points*

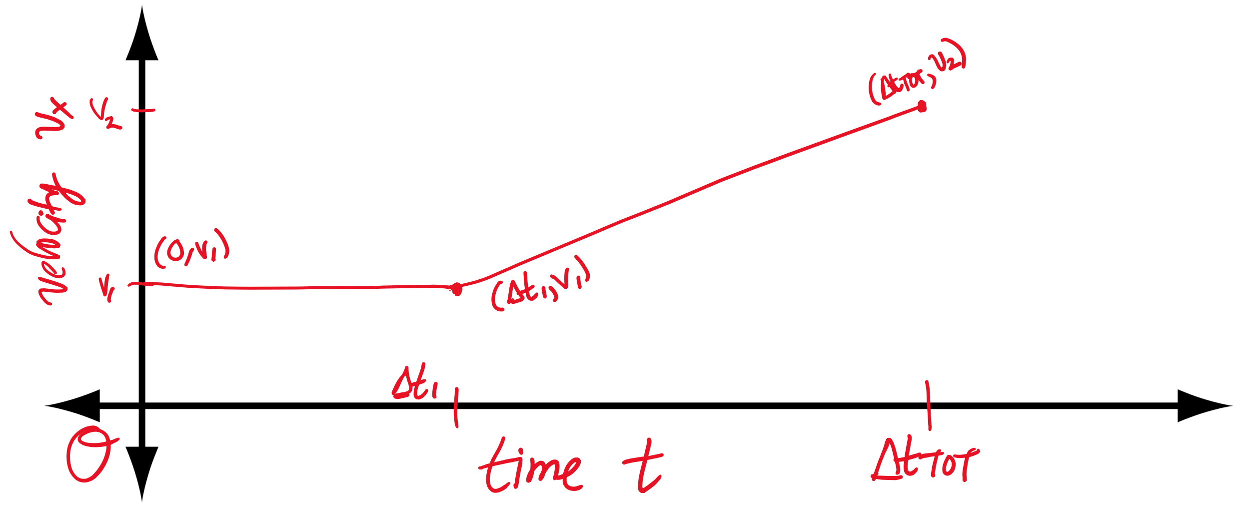
Now suppose the duration Δt_1 is much less than the duration Δt_2. Estimate the x-displacement through which the car travels during the experiment. Express your answer in terms of v_1, v_2, Δt_{TOT}, and fundamental constants, as appropriate.

Briefly explain your reasoning without deriving or manipulating equations.

Correct answer: $\frac{v_1+v_2}{2} \Delta t_{\mathrm{TOT}}$		
For a correct answer and attempt at a consistent justification		1 point
For correct reasoning		1 point
Example earning 2 points: $\frac{v_1+v_2}{2} \Delta t_{\mathrm{TOT}}$. Because Δt_1 is much less than Δt_2, Δt_2 will last for the majority of the total duration Δt_{TOT}. Consequently, the majority of Δt_{TOT} is spent having the car accelerate uniformly from velocity v_1 to velocity v_2. The average velocity for UAM is the arithmetic average of the initial and final velocities. So, the x-displacement, which is the average velocity times the duration, will be $\frac{v_1+v_2}{2} \Delta t_{\mathrm{TOT}}$.		

(b) *2 points*

Now suppose neither time duration Δt_1 nor time duration Δt_2 is much greater than the other, but that they are not necessarily equal. On the axes below, draw a velocity vs. time graph for the car. Label axes and the values of times and velocities at the beginning and end of each phase of motion.



For a horizontal line segment from time $t = 0$ until time $t = \Delta t_1$ at height v_1, with endpoint quantities labeled.	1 point
For a slanted line segment with one endpoint for velocity v_1 at time $t = \Delta t_!$ and another endpoint for velocity v_2 at time $t = \Delta t_{TOT}$ (ok to express Δt_{TOT} as $\Delta t_1 + \Delta t_2$), with endpoint quantities labeled.	1 point

(c) *3 points*

Derive an equation for the x-displacement through which the car travels during the experiment in terms of v_1, v_2, Δt_1, Δt_{TOT}, and fundamental constants, as appropriate. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For using a kinematics principle (equation and/or "signed area under graph" technique) to obtain the x-displacement covered in the first phase of motion $$x_1 = v_1 \Delta t_1$$	1 point
For using a kinematics principle (equation and/or "signed area under graph" technique) to obtain the x-displacement covered in the second phase of motion $$\Delta x_2 = \frac{v_1 + v_2}{2} \Delta t_2$$	1 point
For combining the individual x-displacements from the two phases of motion and eliminating the variable Δt_2 by recognizing $\Delta t_2 = \Delta t_{TOT} - \Delta t_1$ $$\Delta x = v_1 \Delta t_1 + \frac{v_1 + v_2}{2} (\Delta t_{TOT} - \Delta t_1)$$	1 point

(d) *1 point*

Consider the scenario from part (a)(ii), where the duration Δt_1 is much less than the duration Δt_2. Does your equation for the x-displacement of the car from part (c) agree with your reasoning in part (a)(ii)?

___ Yes ___ No

Briefly explain your reasoning by addressing why, according to your equation, the x-displacement becomes (or approaches) the value of a certain expression when Δt_1 is much less than Δt_2.

Correct answer: "Yes"	
Note: "No" is acceptable if the equation is inconsistent with the answer in (a)(ii)	
For valid reasoning that addresses the result in part (c) and the reasoning in part (a)(ii) Example: When Δt_1 is much less than Δt_2, Δt_1 is also much less than Δt_{TOT} and can be sent toward zero, both in the first term $v_1 \Delta t_1$ and in the second term $\frac{v_1+v_2}{2}(\Delta t_{\mathrm{TOT}} - \Delta t_1)$, leaving behind $\frac{v_1+v_2}{2}\Delta t_{\mathrm{TOT}}$.	1 point

(e) *2 points*

In one trial, the car travels through an overall displacement Δx, and the acceleration of the car during the second phase of motion is a_{I}. In a second trial, the car travels through the same overall displacement Δx through the same total time Δt_{TOT} and achieves the same final velocity v_2, but starts out with a slightly greater initial velocity v_1 (still less than v_2). The acceleration of the car during the second phase of motion during the second experiment is a_{II}. How do the two accelerations a_{I} and a_{II} compare to each other?

___ $a_{\mathrm{I}} > a_{\mathrm{II}}$ ___ $a_{\mathrm{I}} = a_{\mathrm{II}}$ ___ $a_{\mathrm{I}} < a_{\mathrm{II}}$

Briefly explain your reasoning. You may use (a) diagram(s) to support your explanation.

Correct answer: $a_{\mathrm{I}} < a_{\mathrm{II}}$ (surprise!) Note: A maximum of 1 point can be earned if an incorrect selection is made.	
For reasoning that merely increasing v_1 while keeping acceleration during the second phase of motion unchanged would, by itself, result in greater displacement	1 point
For reasoning that the duration spent accelerating toward/through speeds higher than $\lvert v_1 \rvert$ must be shortened to reduce the x-displacement covered by the car	1 point

1.1.4 **PLR.** Projectile motion (can be delayed until forces/dynamics)

A cannonball is launched at a slanted direction (partially upwards and partially sidewards) from the edge of a cliff above the ocean below. If the launch speed is doubled, will the time of flight (from launch to splash) be less than doubled, exactly doubled, or more than doubled? Explain your reasoning.

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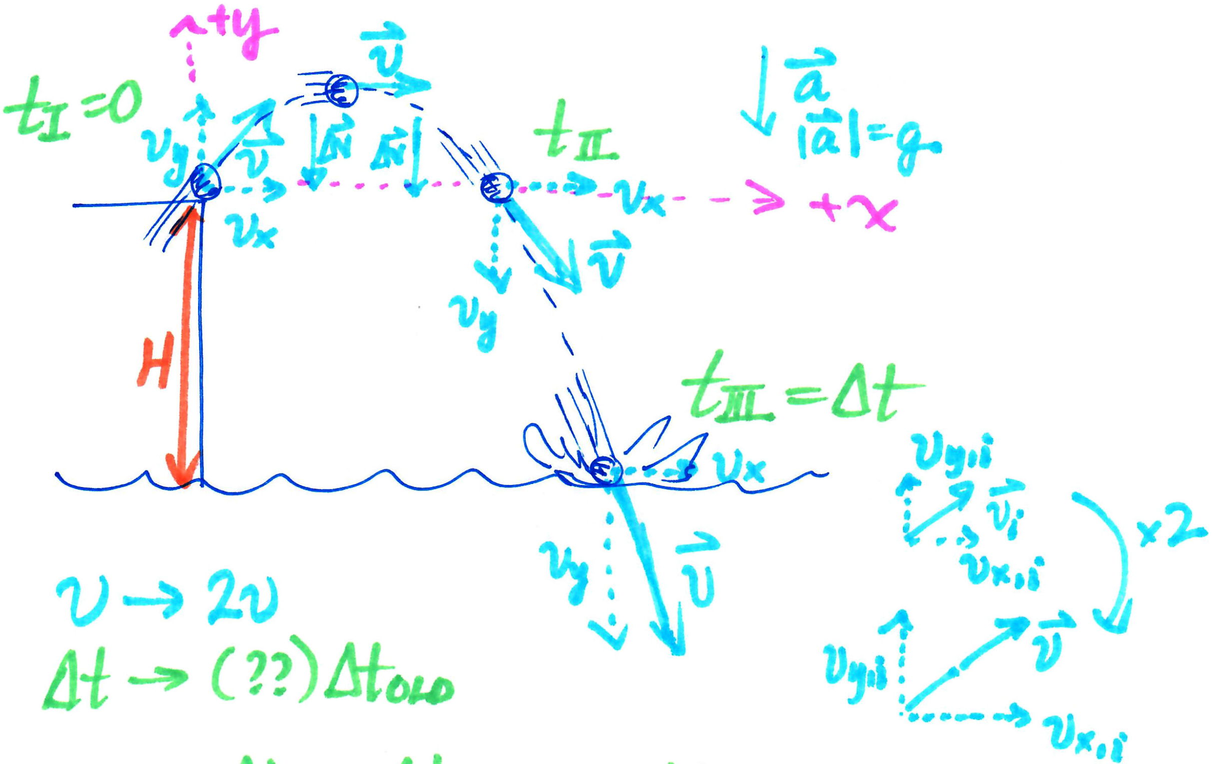
1.1.4.1 Key

A cannonball is launched at a slanted direction (partially upwards and partially sidewards) from the edge of a cliff above the ocean below. If the launch speed is doubled, will the time of flight (from launch to splash) be less than doubled, exactly doubled, or more than double? Explain your reasoning.



$v \to 2v$

$\Delta t \to (??)\Delta t_{old}$

$$\Delta t = \Delta t_{I\to II} + \Delta t_{II\to III}$$

$\uparrow\times 2$

$I \to II$:

$$\Delta y = v_{y,i}\Delta t + \tfrac{1}{2}a_y \Delta t^2$$

$$0 = v_{y,I}\,\Delta t_{I\to II} + \tfrac{1}{2}(-g)\,\Delta t_{I\to II}^2$$

$$0 = \Delta t_{I\to II}\left(v_{y,I} - \tfrac{1}{2}g\,\Delta t_{I\to II}\right)$$

$\Delta t_{I\to 0} = 0$ OR $v_{y,I} = \tfrac{1}{2}g\,\Delta t_{I\to II}$

describes moment of/after launch

$$\Delta t_{I\to II} = \frac{2v_{y,I}}{g}.$$

$$\Delta t_{I\to II} \propto v_{y,I}$$

$\uparrow\times 2 \Leftarrow \uparrow\times 2$

II → III:

$$\Delta y = v_{y,i}\Delta t + \frac{1}{2}a_y \Delta t^2$$

$$-H = v_{y,II}\,\Delta t_{II\to III} + \frac{1}{2}(-g)\Delta t^2_{II\to III}$$

I → II:

$$v_{y,i}^2 + 2a_y\Delta y = v_{y,f}^2$$

$$v_{y,I}^2 + 2(-g)(0) = v_{y,II}^2$$

$$v_{y,I}^2 = v_{y,II}^2$$

$$-v_{y,I} = v_{y,II}$$

→ because on "the way down" at snapshot II.

II → III:

$$-H = (-v_{y,I})\Delta t_{II\to III} - \frac{1}{2}g\,\Delta t^2_{II\to III}$$

$$H = v_{y,I}\,\Delta t_{II\to III} + \frac{1}{2}g\,\Delta t^2_{II\to III}$$

↑×2

$\Delta t_{II\to III}$ must shrink to maintain equality

$$\Delta t = \Delta t_{I\to II} + \Delta t_{II\to III}$$

↑×2 ↓

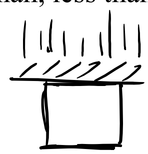
Did not double both terms. Actually shrank second term.

So Δt less than doubles.

1.2 Forces/Newton's laws

1.2.1 **PLR.** Not-proportional reasoning: Normal force on block

In the experiment below, a block near Earth's surface is pressed from above by a movable panel. The block is gaining speed while moving downward with a constant acceleration. The experiment is repeated, but now with the normal force provided by the panel only half as strong as before. Will the magnitude of the resulting acceleration of the block be greater than, less than, or equal to half the original magnitude of acceleration of the block? Explain.



1.2.2 *ExptDes.* Equivalence principle

The equivalence principle states that for a given an object, the inertial mass and the gravitational mass have equal values. Students are asked to test the equivalence principle by working with resources listed in the table below.

Resource	Description
Cart	Has massless wheels that spin without friction
Floor	Smooth, firm horizontal planar surface
Horizontally-mounted motion detector	Tracks horizontal position of object on floor, can be used to produce position-time, velocity-time, and acceleration-time plots on a computer.
Force probe	Displays the strength of the pull force applied by the force probe to whatever object is attached to the force probe's hook
Classmates	Other students can hold and read items for you

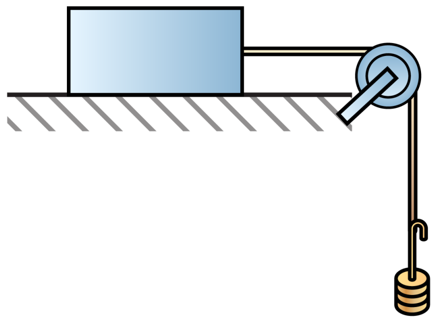
(a) Provide an experimental procedure that could provide data that could be used to test whether the equivalence principle is satisfied by the cart. Give enough detail so that another student could carry out your procedure. Be sure to provide a way to reduce experimental uncertainty.

(b) Describe an analytic procedure that could be applied to the data obtained using the method you described in part (a) to determine whether the equivalence principle is satisfied by the cart. Give enough detail so that another student could perform your analysis.

(c) Using physics principles, explain why the analytic procedure you described in part (b) would work.

1.2.3 **QQT.** Correct false reasoning: Block-string-pendant mass

(a) (11 points, suggested time 22 min 55 sec) In the figure below, a block of mass M on a frictionless, horizontal tabletop is connected to a taut massless string that runs over a frictionless, massless pulley and connects to a slotted mass set of mass M that is initially located a height H above the floor. The block and slotted mass set are released from rest such that the string remains taut and all parts of the system consisting of the block, string, and slotted mass set have equal magnitude of acceleration a_1 until the slotted mass set hits the floor.



A second experiment is planned in which the mass of the slotted mass set will double (now having mass $2M$). A student is asked to predict how the magnitude of system's new acceleration a_2 will compare with the original magnitude of acceleration a_1.

Student: Doubling the mass of the slotted mass set doubles the component of the net force on the system along the direction of acceleration. The block is the portion of the system trying to resist being accelerated, so keeping the same block means keeping the inertial mass of the system the same. According to Newton's 2nd law, acceleration is proportional to net force and inversely proportional to inertial mass, so the magnitude of the acceleration will increase, specifically, $a_2 = 2a_1$.

(a)

i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

(b) Use quantitative reasoning, including equations as needed, to develop an expression for the new magnitude of acceleration a_2 of the system. Express your answer in terms of a_1.

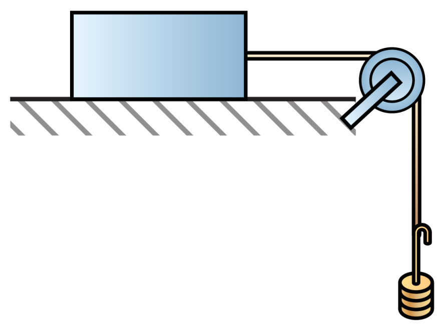
(c)

i. Explain how any correct aspects of the student's reasoning identified in part (a) are expressed by your mathematical relationships in part (b). Your response should refer to the relationships you wrote in part (b), not only to the final answer you obtained by manipulating those relationships.

ii. Explain how your relationships in part (b) correct any incorrect aspects of the student's reasoning identified in part (a). Your response should refer to the relationships you wrote in part (b), not only to the final answer you obtained by manipulating those relationships.

1.2.3.1 Key

In the figure below, a block of mass M on a frictionless, horizontal tabletop is connected to a taut massless string that runs over a frictionless, massless pulley and connects to a slotted mass set of mass M that is initially located a height H above the floor. The block and slotted mass set are released from rest such that the string remains taut and all parts of the system consisting of the block, string, and slotted mass set have equal magnitude of acceleration a_1 until the slotted mass set hits the floor.



A second experiment is planned in which the mass of the slotted mass set will double (now having mass $2M$). A student is asked to predict how the magnitude of system's new acceleration a_2 will compare with the original magnitude of acceleration a_1.

Student: Doubling the mass of the slotted mass set doubles the component of the net force on the system along the direction of acceleration. The block is the portion of the system trying to resist being accelerated, so keeping the same block means keeping the inertial mass of the system the same. According to Newton's 2nd law, acceleration is proportional to net force and inversely proportional to inertial mass, so the magnitude of the acceleration will increase, specifically, $a_2 = 2a_1$.

(a)

i. Which aspects of the student's reasoning, if any, are correct? Explain how you arrived at your answer.

2 points total

1 point Student is correct to claim that "Doubling the mass of the slotted mass set doubles the component of the net force on the system along the direction of acceleration" because component of net force along direction of acceleration equals weight of hanging slotted mass set and weight of slotted mass equals the product of the mass set's mass and g.

1 point Student correctly cites Newton's 2nd law and provides a correct qualitative conclusion that the magnitude of system acceleration will increase. The factor by which the component of the net force increases (doubles) is greater than the factor by which the total system inertial mass increases, so that magnitude of system acceleration does, indeed, increase.

ii. Which aspects of the student's reasoning, if any, are incorrect? Explain how you arrived at your answer.

2 points total

1 point Student incorrectly claims that the "block is the portion of the system trying to resist being accelerated, so keeping the same block means keeping the inertial mass of the system the same". Actually, all of the inertial mass of the block and slotted mass *together* contributes to the total inertial mass of the system, so keeping the same block while increasing the mass of the slotted mass set actually increases the total inertial mass of the system.

1 point Because the student incorrectly thinks that the total inertial mass of the system remains unchanged while the component of the net force along the direction of acceleration doubles, the student incorrectly concludes that the factor of increase of the magnitude of the system's acceleration is two. Because the inertial mass of the system also increases (though not by doubling), the factor by which the system's acceleration is multiplied is less than two.

(b) Use quantitative reasoning, including equations as needed, to develop an expression for the new magnitude of acceleration a_2 of the system. Express your answer in terms of a_1.

3 points total

1 point For using Newton's 2nd law to conclude that $a_2 = \frac{4}{3}a_1$

1 point Component of net force in direction of acceleration increases by factor of two because mass of slotted mass set doubles

1 point Total system inertial mass increases to 150% original value because the mass of the slotted mass set was doubled while the mass of the block stayed the same

(c)

i. Explain how any correct aspects of the student's reasoning identified in part (a) are expressed by your mathematical relationships in part (b). Your response should refer to the relationships you wrote in part (b), not only to the final answer you obtained by manipulating those relationships.

2 points total

1 point Identifying which specific mathematical expressions and/or manipulations in part (b) represent doubling of component of net force along direction of acceleration

1 point Identifying which specific mathematical expressions and/or manipulations in part (b) represent increase in acceleration

i. Explain how your relationships in part (b) correct any incorrect aspects of the student's reasoning identified in part (a). Your response should refer to the relationships you wrote in part (b), not only to the final answer you obtained by manipulating those relationships.

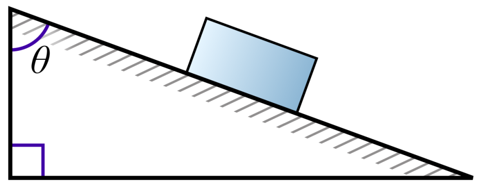
2 points total

1 point Identifying which specific mathematical expressions and/or manipulations in part (b) express that the total inertial mass is increased (by a factor of $\frac{3}{2}$) rather than remaining unchanged because both the unchanged mass of the block and the doubled mass of the slotted mass set contribute to the total inertial mass of the system

1 point Identifying which specific mathematical expressions and/or manipulations in part (b) express that the increase in the magnitude of the system's acceleration is less than a doubling

1.2.4 **PLR.** Labeling quantities in unusual places: Inclined ramp
(Short sample question):

A block is sliding on a fixed frictionless inclined plane having an angle θ as illustrated in the figure below. The acceleration of the block has magnitude a. If the angle θ is decreased, will the magnitude of the block's acceleration increase, decrease, or stay the same? Explain your reasoning.



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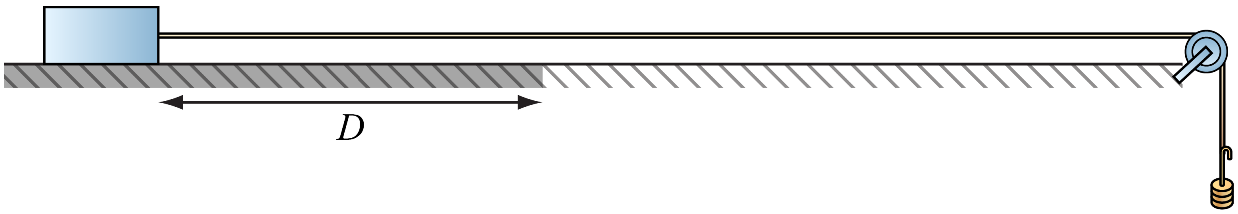
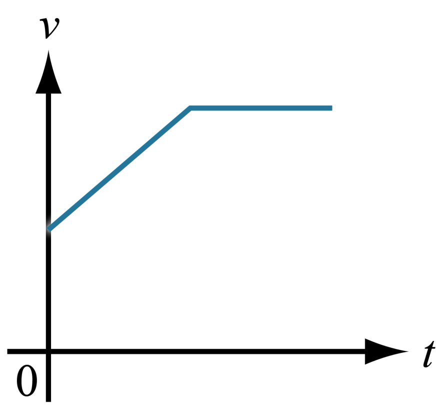
1.2.5 *Explaining a choice:* Block, half-rough surface, pulley

In the figure below, a block of mass m is connected to a massless string draped over a massless, frictionless pulley. The other end of the string is connected to a slotted mass set, also of mass m. The block is on the rough portion (darker shading) of a horizontal tabletop, and the coefficients of kinetic and static friction between the block and the rough portion of the table are $\mu_K = \mu_S = 2.0$. The block is initially skidding to the right with speed $v = 10.\frac{\text{m}}{\text{s}}$ at a distance $D = 7.5$ m away from the location where the rough portion of the tabletop joins the frictionless portion of the tabletop (lighter shading).

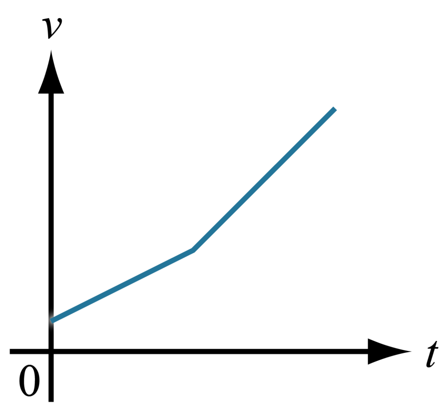


A video recording is analyzed to obtain a speed-time plot for the block. The tabletop is long, and the block does not crash into the pulley before the video recording is stopped. Which one of the following graphs represents the speed-time graph for the block? Explain how you arrived at your choice.

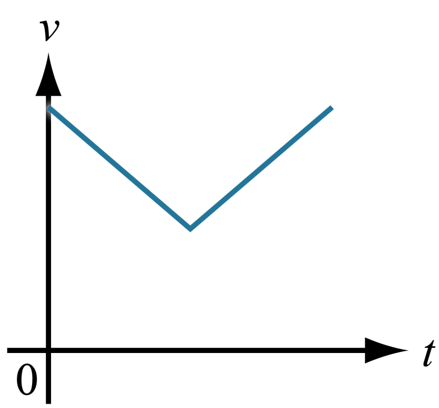
(A)



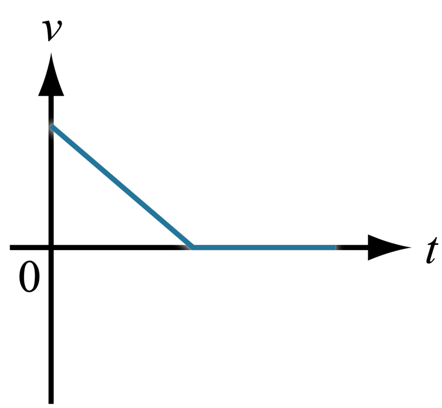
(B)



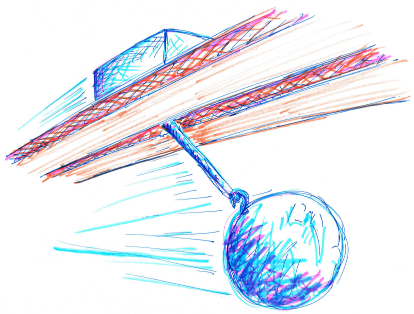
(C)



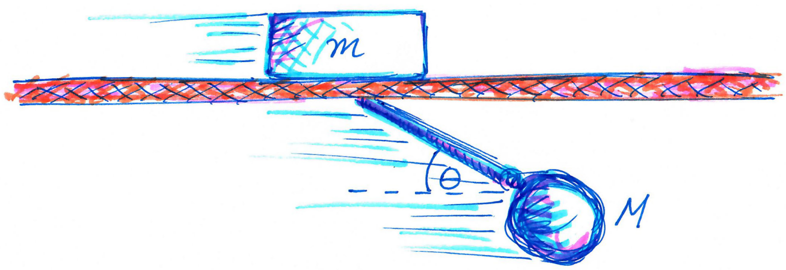
(D)



1.2.6 **QQT.** Unfamiliar scenario: Block-String-Leading ball
As shown below, a block of mass m on two rough horizontal planks is attached to a massless string, the other end of which is connected to a sphere of mass M. The coefficient of kinetic friction between the block and planks is μ_K. It is possible (perhaps after some practice trials) to launch the block, string, and sphere so that the angle θ between the tight string and the horizontal has a constant value during the time interval when the block, string, and sphere are still moving together and have not yet come to rest. The acceleration of the block-string-sphere system before coming to rest has magnitude a.



Perspective view



Side view

(a) Suppose that the experiment is repeated, but now with mass M doubled.

i. Does the magnitude a of the acceleration of the block-string-sphere system increase, decrease, or stay the same?

__ Increase __ Decrease __ Remain the same

Qualitatively explain your reasoning without algebraic manipulations (you can write, cite, and describe features of algebraic equations, but do not merely write, “I solved the system of 3 equations over there, and the final expression shows that …”).

ii. Does the angle θ increase, decrease, or stay the same?

__ Increase __ Decrease __ Remain the same

Qualitatively explain your reasoning without algebraic manipulations (you can write, cite, and describe features of algebraic equations, but do not merely write, “I solved the system of 3 equations over there, and the final expression shows that …”).

(b)

i. Mathematically derive an expression for the magnitude a of the acceleration in terms of m, M, μ_K, and fundamental constants, as needed.

ii. Mathematically derive an expression for the angle θ in terms of m, M, μ_K, and fundamental constants, as needed.

A class of students attempted to perform the experiment described at the beginning of this problem. There was only one set of equipment (block, string, sphere, planks, and video recording equipment), so students had to share (only one group was able to perform the experiment at a time; groups of students had to wait their turn to use the equipment). The table below shows the angle θ measured by each group using video analysis.

Group	Time group finished experiment	Angle θ ($\pm 2°$)
1	10:02 am	22
2	10:07 am	21
3	10:12 am	24
4	10:18 am	25
5	10:22 am	28
6	10:30 am	33
7	10:37 am	31

(c)

i. The students notice that the angle θ is mostly increasing from group to group. One student accuses the class of sabotaging the equipment. The student proposes that each group shaved some mass off of the block, causing the block to be pressed against the planks less and less tightly as the class period continued, which in turn reduced the strength of the kinetic friction force applied by the planks on the block. The student argues that weaker friction on the block resulted in less acceleration of the block-string-sphere system, and, thus, a more vertically oriented string. Is the student's accusation consistent with the mathematical work in part (b)? Explain.

ii. Provide an alternative reason for the trend of increasing angle θ. Use your mathematical result in part (b) to support your alternative reason.

1.2.7 *ExptDes.* Test false claim: Static friction

A student claims that the maximum static friction force f_S^{MAX} that can be provided before two surfaces slip increases in proportion to the normal force between the surfaces until reaching a maximum strength and that the f_S^{MAX} then has this same maximum strength for all greater magnitudes of normal force.

(a) Describe an experimental setup that could be used to test the student's claim for a wooden block and a wooden board. You may use any equipment ordinarily available in a typical physics classroom laboratory. Include a diagram of your setup.

(b) How could you use the experimental setup you described in (a) to obtain data to test the student's claim? Be sure to indicate which equipment is used to measure each relevant quantity. Provide enough detail so that another student could carry out your experimental procedure. Provide a method to reduce uncertainty.

(c) How could you analyze the data you would obtain according to the procedure your described in (b) to test the student's claim? Include a way to take measurement uncertainty into account.

(d) Using physical principles and reasoning, explain how you know that your analysis method could be used to test the student's claim.

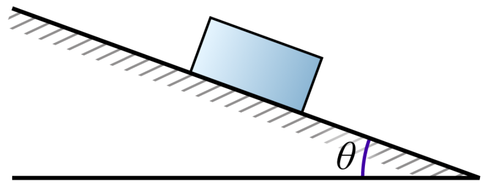
The student performs an experiment using a wood block and wooden board and obtains the data in the table below.

Normal force (± 0.02 N)	Maximum static friction force (± 0.1 N)
2.00	1.6
4.00	3.0
8.00	5.9
12.00	8.9
16.00	12.0
20.00	15.1

(e) Are the data in the table consistent with the student's claim? Justify your answer.

1.2.8 *Warm-up/short* **PLR**s. Kinetic friction, inclined ramp

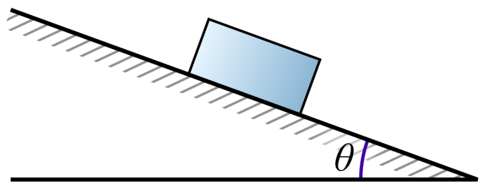
In the figure below, a block is sliding on an inclined plane having angle of inclination θ. Because "whooshies" are not drawn, it might be the case that the block is sliding up the plane. It also might be the case that the block is sliding down the plane.



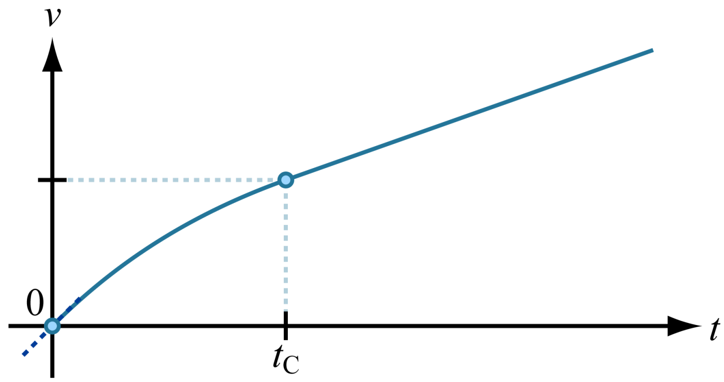
(b) If the plane is frictionless, is the magnitude of the acceleration of the block greater while sliding up the plane, greater while sliding down the plane, or equal for both cases? Explain your reasoning.

(c) Assuming now that there is friction between the plane and the box, of equal strength while the box slides up the plane and while the box slides down the plane, is the magnitude of the acceleration of the block greater while sliding up the plane, greater while sliding down the plane, or equal for both cases? Explain your reasoning.

1.2.9 **PLR.** Non-proportional reasoning with trigonometry: Block on accelerating wedge
In the figure below, the frictionless inclined plane has angle of inclination θ and is accelerating, purely horizontally to the right, with an acceleration of magnitude a. As a result, the box remains in contact with the plane at a constant height (rather than sliding up or down along the plane). In a second experiment, the magnitude of acceleration a is doubled. In order for the box to remain in contact with the plane at a constant height (again, rather than sliding up or down along the plane), will the angle of inclination need to more than double, less than double, or exactly double? Explain your reasoning.



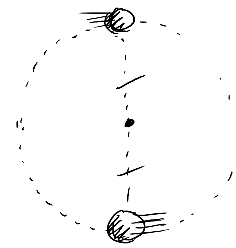
1.2.10 **PLR**. Fluid drag.



At time $t = 0$, a massive object is released from rest and falls toward the Earth through a fluid medium in which drag is not negligible. The graph of the object's speed v as a function of time t is shown above. The graph consists of an initial portion from time $t = 0$ to time $t = t_C$ that is curved (concave down) and a second portion from time $t = t_C$ onward that is a straight segment. A student claims that the strength F_{DRAG} of the drag force exerted by the fluid on the object increases whenever the speed v of the object through the fluid increases. In a clear-coherent paragraph-length response that incorporates physics principles, explain whether the graph supports the student's claim.

1.3 Dynamics (with circular motion, gravitation, orbits)

1.3.1 **QQT.** Explaining scaling: Orbits



A pair of stars, each having mass m, orbit a point midway between them. The distance between the stars is ℓ.

(a) If the mass of each star is increased by the same factor, how will the orbital speed of each star be affected? Indicate increase, decrease, or remain the same.

___ Increase ___ Decrease ___ Remain the same

Justify your answer.

(b) Derive an expression for the orbital speed v of each star in terms of m, ℓ, and fundamental constants, as needed.

1.3.2 *DataAnalysis.* Deviation from Kepler's 3rd law

An astronomer discovers a new planet that is circularly orbited by four moons. The astronomer obtains the orbital radii and periods for the four moons in the table below.

Moon	Orbital radius R (m) $\pm 3 \times 10^7$	Orbital period T (s) $\pm 2 \times 10^3$		
A	6.2×10^8	2.66×10^5		
B	9.2×10^8	4.80×10^5		
C	1.07×10^9	4.26×10^5		
D	1.20×10^9	5.06×10^5		

One of the astronomer's colleagues claims that practically all the mass in the vicinity of the newly discovered planet and its four moons is concentrated in a point-like fashion in the center of the planet itself.

(a) Provide a method for analyzing the data to test the colleague's claim. Be sure to provide enough detail so that another physics student could carry out your analysis procedure. Provide a method to take experimental uncertainty into account.

(b) Use physics principles to explain why the analysis method you described in part (a) can be used to test the colleague's claim.

(c) Carry out the method you described in part (a) and report whether the colleague's claim is correct. You may use the gridded space below and/or empty space in the data table in the problem statement if needed for your analysis method.

1.3.2.1 Key

An astronomer discovers a new planet that is circularly orbited by four moons. The astronomer obtains the orbital radii and periods for the four moons in the table below.

Moon	Orbital radius R (m) $\pm 3\times10^{7}$	Orbital period T (s) $\pm 2\times10^{3}$	R^3 (m^3)	T^2 (s^2)
A	6.2×10^{8}	2.66×10^{5}	2.38×10^{26}	7.05×10^{10}
B	9.2×10^{8}	4.80×10^{5}	7.79×10^{26}	2.30×10^{11}
C	1.07×10^{9}	4.26×10^{5}	1.22×10^{27}	1.81×10^{11}
D	1.20×10^{9}	5.06×10^{5}	1.73×10^{27}	2.56×10^{11}

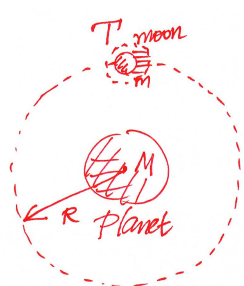
One of the astronomer's colleagues claims that practically all the mass in the vicinity of the newly discovered planet and its four moons is concentrated in a point-like fashion in the center of the planet itself.

(a) Provide a method for analyzing the data to test the colleague's claim. Be sure to provide enough detail so that another physics student could carry out your analysis procedure. Provide a method to take experimental uncertainty into account.

- Add columns in table for R^3 (m^3) and T^2 (s^2).
- Estimate uncertainty in T^2 by recalculating T^2 for at least one of the moons using a value of T that differs from the reported value of T by an amount equal to the uncertainty in T.
- Estimate uncertainty in R^3 by recalculating R^3 for at least one of the moons using a value of R that differs from the reported value of R by an amount equal to the uncertainty in R.
- Make scatterplot of T^2 vs. R^3 with error bars.
- If scatterplot is well fit by a line of best fit (LBoF) forced to pass through the origin, then the data are consistent with the colleague's claim. If the scatterplot is not well fit by a LBoF forced to pass through the origin, then the data are inconsistent with the colleague's claim.
- A scatterplot is well fit by an LBoF if the LBoF threads/close-to-threads all of the error bars.

(b) Use physics principles to explain why the analysis method you described in part (a) can be used to test the colleague's claim.

According to colleague's claim, practically all the mass in the vicinity of the planet and its moons is concentrated at the center of the planet. Everything else in the vicinity must have extremely low (or no) mass, so each moon is only substantially gravitationally attracted by the planet itself.



$$ma_{\mathrm{IN}} = \sum F_{\mathrm{IN}}$$

$$\frac{mv^2}{R} = F_{\mathrm{G,PLANET\to moon}}$$

$$\frac{\cancel{m}v^2}{R} = G\frac{M\cancel{m}}{R^2}$$

$$\frac{\left(\frac{2\pi R}{T}\right)^2}{R} = G\frac{M}{R^2}$$

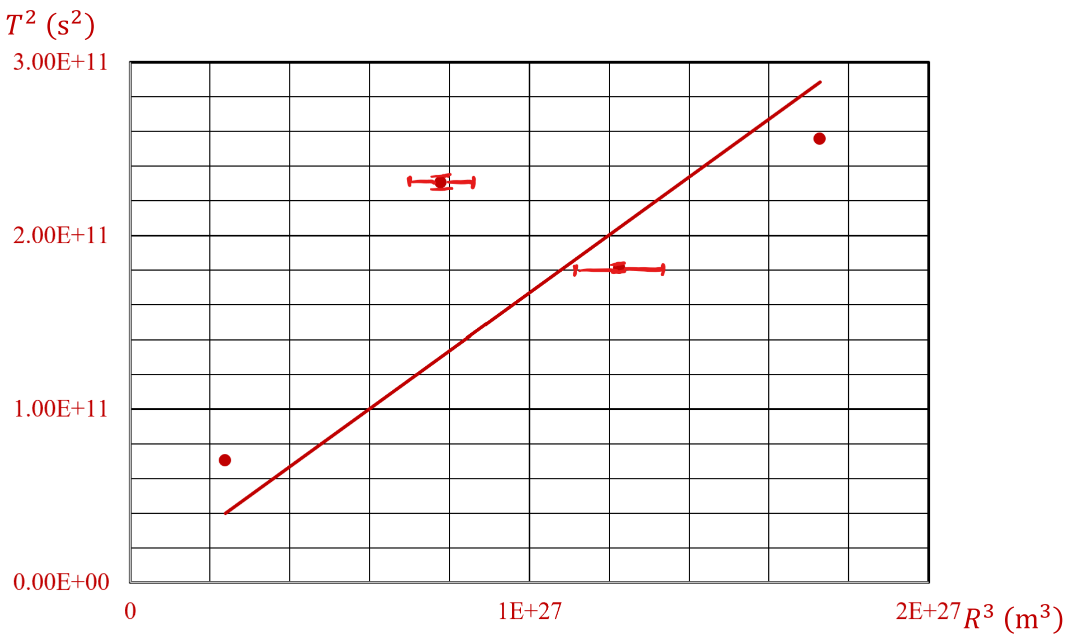
$$T^2 = \frac{4\pi^2}{GM}R^3$$

According to N2L for uniform circular motion, $ma_{\text{IN}} = \sum F_{\text{IN}}$ for each moon (let m be mass of a moon). The radially inward acceleration is $a_{\text{IN}} = v^2/R$, and the tangential speed in terms of period is $v = \frac{2\pi R}{T}$. The net radially inward force is provided exclusively by the gravitational force the planet exerts on that moon, $F_{\text{G,PLANET}\to\text{moon}} = G\frac{Mm}{r^2}$, so the colleague's claim leads to the relationship

$$T^2 = \frac{4\pi^2}{GM}R^3$$

which has slope-intercept form $y = \text{slope}\, x + b$ with T^2 playing the role of y, $\frac{4\pi^2}{GM}$ playing the role of slope, R^3 playing the role of x, and 0 playing the role of b, the y-coordinate of the y-intercept. Thus, the colleague's claim can be tested by testing whether a scatterplot of T^2 vs. R^3 is consistent with a linear relationship that passes through the origin.

(c) Carry out the method you described in part (a) and report whether the colleague's claim is correct. You may use the gridded space below and/or empty space in the data table in the problem statement if needed for your analysis method.



Estimate uncertainty in T^2 for moon B:

$$T^2\left(\text{using } T = \underbrace{4.80\times10^5}_{\text{reported } T} + \underbrace{2\times10^3}_{\text{uncertainty in } T}\right) = 2.33\times10^{11}\ \text{s}^2$$

Estimate uncertainty in R^3 for moon B:

$$R^3\left(\text{using } R = \underbrace{9.2\times10^8}_{\text{reported } R} + \underbrace{3\times10^7}_{\text{uncertainty in } R}\right) = 8.57\times10^{26}\ \text{m}^3$$

(Also estimated errors and drew error bars for moon C).

LBoF through origin does not thread/almost thread all of the error bars (in particular, LBoF very far away from threading error bars for moon B). So, data are inconsistent with the colleague's claim. Thus, the colleague's claim is incorrect.

1.4 Impulse-momentum

1.4.1 Warm-up **PLR**.

Two blocks, of masses m and M, are on a horizontal frictionless track. The block of mass m is initially traveling with initial speed v_1 before crashing into and sticking to the block of mass M, which was initially at rest. In a clear, coherent paragraph-length response that makes use of physical principles, explain whether the final speed v_2 of the stuck-together blocks after the crash is greater than, less than, or the same as v_1.

___ $v_2 > v_1$ ___ $v_2 < v_1$ ___ $v_2 = v_1$

1.4.2 **PLR.** False-reasoning task: Momentum.
On a horizontal, frictionless track, cart A is launched toward initially stationary identical cart B. The two carts immediately stick upon contact. Charlie tried to use conservation of momentum to determine the final speed of the stuck-together carts, but Charlie misremembered the formula for the x-momentum as $p_x = \frac{1}{2}mv_x^2$. Indicate whether the speed of the stuck-together carts Charlie calculated was greater than, less than, or equal to the speed of the stuck-together carts Charlie should have calculated.

____ Greater than ____ Less than ____ Equal

Explain your reasoning.

1.4.3 *ExptDes.* Testing a false claim: Momentum

("25" min, actually probably much longer) Students are provided with two carts that they can collide and a track on which the carts move with practically no friction. Students are allowed to use equipment ordinarily found in a high school physics laboratory except for instruments for measuring forces. Alice claims that there is a special magnitude of momentum P_C so that as long as the total pre-collision momentum of the system's magnitude is less than P_C, the total post-collision momentum will equal the total pre-collision momentum but whenever, instead, the total pre-collision momentum of the system is greater than or equal to P_C the total post-collision momentum will have the same direction as the total pre-collision momentum but have magnitude equal to P_C.

(a) Describe an experimental procedure that can be used to test Alice's claim. You can include a labeled diagram of your setup. Indicate which equipment is used to make each measurement. Give enough detail so that another student could carry out your procedure. Specify how you will minimize and estimate experimental uncertainty.

(b) Describe a method that can be used to analyze the data you would obtain in part (a) above in order to test Alice's claim. Provide enough detail so that another student could carry out your analysis. Take into account experimental uncertainty.

(c) Explain how you know that your analytic method can be used to test Alice's claim.

(d) In a later class, students collided two carts of mass $m_1 = 0.30$ kg and $m_2 = 0.53$ kg on a horizontal, frictionless track and obtained the following (x-)velocity data. The velocities of cart k before and after the collision are $v_{k,i}$ and $v_{k,f}$, respectively.

	Pre-collision		Post-collision			
Trial	$v_{1,i}$ $\left(\frac{\text{m}}{\text{s}}\right)$	$v_{2,i}$ $\left(\frac{\text{m}}{\text{s}}\right)$	$v_{1,f}$ $\left(\frac{\text{m}}{\text{s}}\right)$	$v_{2,f}$ $\left(\frac{\text{m}}{\text{s}}\right)$		
1	0.21	0.00	0.10	0.07		
4	1.32	−0.43	−0.25	0.47		
3	0.84	−0.12	−0.32	0.54		
2	0.39	0.20	−0.30	0.60		

Taking into account that all velocities were measured with an uncertainty of about $\pm 0.02 \frac{\text{m}}{\text{s}}$, are the data consistent with Alice's claim? Justify your answer. You may use the blank columns in the table above, if needed for your analysis.

1.4.3.1 Key

Students are provided with two carts that they can collide and a track on which the carts move with practically no friction. Students are allowed to use equipment ordinarily found in a high school physics laboratory except for instruments for measuring forces. Alice claims that there is a special magnitude of momentum P_C so that as long as the total pre-collision momentum of the system's magnitude is less than P_C, the total post-collision momentum will equal the total pre-collision momentum but whenever, instead, the total pre-collision momentum of the system is greater than or equal to P_C the total post-collision momentum will have the same direction as the total pre-collision momentum but have magnitude equal to P_C.

(a) Describe an experimental procedure that can be used to test Alice's claim. You can include a labeled diagram of your setup. Indicate which equipment is used to make each measurement. Give enough detail so that another student could carry out your procedure. Specify how you will minimize and estimate experimental uncertainty.

1 point	Setup/procedure for configuring/launching equipment that can viably be used to test Alice's claim
1 point	Viable set of measured quantities; for every individual measured quantity, the associated instrument used to make the measurement is clearly indicated
1 point	Reasonable method for minimizing and estimating experimental uncertainty

(b) Describe a method that can be used to analyze the data you would obtain in part (a) above in order to test Alice's claim. Provide enough detail so that another student could carry out your analysis. Take into account experimental uncertainty.

1 point	Specify computation of $m_1v_{1,i} + m_2v_{2,i}$ and $m_1v_{1,f} + m_2v_{2,f}$
1 point	Analytic method for testing whether there exists a magnitude P_C so that when $\lvert m_1v_{1,i} + m_2v_{2,i}\rvert < P_\mathrm{C}$, $m_1v_{1,i} + m_2v_{2,i} = m_1v_{1,f} + m_2v_{2,f}$ and when $\lvert m_1v_{1,i} + m_2v_{2,i}\rvert \geq P_\mathrm{C}$, $m_1v_{1,i} + m_2v_{2,i}$ and $m_1v_{1,f} + m_2v_{2,f}$ have the same sign, but $\lvert m_1v_{1,f} + m_2v_{2,f}\rvert = P_\mathrm{C}$ (and declaring Alice's claim not rejected if such a P_C can be found or declaring Alice's claim rejected if such a P_C cannot be found)
1 point	Specify a reasonable method for using uncertainties (e.g. in velocities) to estimate uncertainties in $m_1v_1 + m_2v_2$ and indicate reasonable way to use uncertainties in $m_1v_1 + m_2v_2$ when making comparisons involving values of $m_1v_1 + m_2v_2$

Make a table of values for each trial:

	Pre-collision		Post-collision	
Trial	$v_{1,i}\ \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$	$v_{2,i}\ \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$	$v_{1,f}\ \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$	$v_{2,f}\ \left(\frac{\mathrm{m}}{\mathrm{s}}\right)$
1				
etc.				

Fill a new column with computed values for $m_1v_{1,i} + m_2v_{2,i}$. Make a second table with rows re-sorted so that values of the magnitude of $m_1v_{1,i} + m_2v_{2,i}$ now increases from top row to bottom row (or use original table if magnitude of $m_1v_{1,i} + m_2v_{2,i}$ already increased from top row to bottom row). Fill a new column with computed value for $m_1v_{1,f} + m_2v_{2,f}$. Ask whether a value P_C can be identified so that every row (if any) in the table with $\lvert m_1v_{1,i} + m_2v_{2,i}\rvert < P_\mathrm{C}$ has a

value of $m_1 v_{1,f} + m_2 v_{2,f}$ equal to $m_1 v_{1,i} + m_2 v_{2,i}$ and every row (if any) in the table with a magnitude of $|m_1 v_{1,i} + m_2 v_{2,i}| \geq P_C$ has a value of $m_1 v_{1,f} + m_2 v_{2,f}$ with the same sign as $m_1 v_{1,i} + m_2 v_{2,i}$ and with $|m_1 v_{1,f} + m_2 v_{2,f}| = P_C$. If yes, Alice's claim is not rejected. If not, Alice's claim is rejected.
Throughout the analysis above, consider two quantities equal within experimental uncertainty so long as the amount by which the quantities differ is roughly no more than the size of the uncertainty in either quantity (e.g. uncertainty in $m_1 v_{1,i} + m_2 v_{2,i}$ estimated by substituting "slightly incorrect" velocity value/s—with incorrect values and observed values differing by roughly the uncertainty of the velocities—and re-computing $m_1 v_{1,i} + m_2 v_{2,i}$ to see how much $m_1 v_{1,i} + m_2 v_{2,i}$ changed). Otherwise, consider the two quantities unequal given experimental uncertainty.

(c) Explain how you know that your analytic method can be used to test Alice's claim.

1 point	Statement of definition of momentum $p_x = mv_x$
1 point	Indication of understanding that analysis of momenta can be accomplished by analysis of mv
1 point	Explicitly concluding that analytic method for testing whether there exists a magnitude P_C so that when $|m_1 v_{1,i} + m_2 v_{2,i}| < P_C$, $m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$ and when $|m_1 v_{1,i} + m_2 v_{2,i}| \geq P_C$, $m_1 v_{1,i} + m_2 v_{2,i}$ and $m_1 v_{1,f} + m_2 v_{2,f}$ have the same sign, but $|m_1 v_{1,f} + m_2 v_{2,f}| = P_C$ (and declaring Alice's claim not rejected if such a P_C can be found or declaring Alice's claim rejected if such a P_C cannot be found) achieves the same result as testing whether there exists a magnitude P_C so that when $|\sum p_i| < P_C$, $\sum p_i = \sum p_f$ and when $|\sum p_i| \geq P_C$, $\sum p_f$ and $\sum p_i$ have the same sign, but $|\sum p_f| = P_C$ (and declaring Alice's claim not rejected if such a P_C can be found or declaring Alice's claim rejected if such a P_C cannot be found).

Alice's claim is about total momentum being constant or not-constant. What does "constant" mean?

Alice's claim:	
When $|\sum p_i| < P_C$	When $|\sum p_i| \geq P_C$
$\sum p_i = \sum p_f$	$\sum p_f$ & $\sum p_i$ have same sign $|\sum p_f| = P_C$
Most apparatus do not literally measure momentum, but, instead measure quantities like mass and velocity. I need a way to connect measurable quantities with p.	
$p_x = mv_x$	
When $|m_1 v_{1,i} + m_2 v_{2,i}| < P_C$	When $|m_1 v_{1,i} + m_2 v_{2,i}| \geq P_C$
$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$	$m_1 v_{1,i} + m_2 v_{2,i}$ & $m_1 v_{1,f} + m_2 v_{2,f}$ have same sign $|m_1 v_{1,f} + m_2 v_{2,f}| = P_C$
☹ Now I need to write out the English sentences.	

If it were magically possible to directly measure the total momentum of the two cart system, testing Alice's claim would be achieved by assessing whether there could be identified a magnitude P_C so that when $|\sum p_i| < P_C$, $\sum p_i = \sum p_f$ and when $|\sum p_i| \geq P_C$, $\sum p_f$ and $\sum p_i$ have the same sign, but $|\sum p_f| = P_C$ (and declaring Alice's claim not rejected if such a P_C can be found or declaring Alice's claim rejected if such a P_C cannot be found). According to the definition of momentum ($p_x = mv_x$), the x-momentum of an object equals the product of the object's mass and x-velocity. So, testing Alice's claim can be indirectly accomplished by replacing every object's p_x with mv_x above and testing, as prescribed in my response to part (b), whether a value P_C can be identified so that when $|m_1 v_{1,i} + m_2 v_{2,i}| < P_C$, $m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$ and when $|m_1 v_{1,i} + m_2 v_{2,i}| \geq P_C$, $m_1 v_{1,i} + m_2 v_{2,i}$ and $m_1 v_{1,f} + m_2 v_{2,f}$ have the same sign, but $|m_1 v_{1,f} + m_2 v_{2,f}| = P_C$ (and declaring Alice's claim not rejected if such a P_C can be found or declaring Alice's claim rejected if such a P_C cannot be found).

Expected wrong answer and comment: It is expected that some students will write responses that read, roughly, "According to the impulse-momentum theorem, the total momentum of the carts will be constant because the track is frictionless and so the net horizontal force on the system is zero. Thus, $m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$."

This wrong answer implicitly shows knowledge that $p_x = mv_x$; this implicit knowledge is good. However, this wrong answer did not explicitly explain **that** knowledge of $p_x = mv_x$ leads to the conclusion that testing whether there exists a magnitude P_C so that when $|m_1 v_{1,i} + m_2 v_{2,i}| < P_C$, $m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$ and when $|m_1 v_{1,i} + m_2 v_{2,i}| \geq P_C$, $m_1 v_{1,i} + m_2 v_{2,i}$ and $m_1 v_{1,f} + m_2 v_{2,f}$ have the same sign, but $|m_1 v_{1,f} + m_2 v_{2,f}| = P_C$ (and declaring Alice's claim not rejected if such a P_C can be found or declaring Alice's claim rejected if such a P_C cannot be found) achieves the same result as testing whether there exists a magnitude P_C so that when $|\sum p_i| < P_C$, $\sum p_i = \sum p_f$ and when $|\sum p_i| \geq P_C$, $\sum p_f$ and $\sum p_i$ have the same sign, but $|\sum p_f| = P_C$ (and declaring Alice's claim not rejected if such a P_C can be found or declaring Alice's claim rejected if such a P_C cannot be found). Additionally, stating a conclusion that $m_1 v_{1,i} + m_2 v_{2,i}$ is supposed to be always equal to $m_1 v_{1,f} + m_2 v_{2,f}$ for the given experimental apparatus doesn't look really good in this written response. Problem parts (a), (b), and (c) are not asking about whether there are ways for us to use theory to *expect* whether total momentum is constant *before* we ever obtain data. Problem parts (a), (b), and (c) are, instead, asking about how we can use experimental data using the given apparatus to generate knowledge to test Alice's claim.

(d) In a later class, students collided two carts of mass $m_1 = 0.30$ kg and $m_2 = 0.53$ kg on a horizontal, frictionless track and obtained the following (x-)velocity data. The velocities of cart k before and after the collision are $v_{k,i}$ and $v_{k,f}$, respectively.

	Pre-collision		Post-collision			
Trial	$v_{1,i}$ $\left(\frac{m}{s}\right)$	$v_{2,i}$ $\left(\frac{m}{s}\right)$	$v_{1,f}$ $\left(\frac{m}{s}\right)$	$v_{2,f}$ $\left(\frac{m}{s}\right)$	$m_1 v_{1,i} + m_2 v_{2,i}$ $\left(\text{kg}\cdot\frac{m}{s}\right)$	$m_1 v_{1,f} + m_2 v_{2,f}$ $\left(\text{kg}\cdot\frac{m}{s}\right)$
1	0.21	0.00	0.10	0.07	0.063	0.067
4	1.32	−0.43	−0.25	0.47	0.168	0.174
3	0.84	−0.12	−0.32	0.54	0.188	0.190
2	0.39	0.20	−0.30	0.60	0.223	0.228

Taking into account that all velocities were measured with an uncertainty of about $\pm 0.02\,\frac{\text{m}}{\text{s}}$, are the data consistent with Alice's claim? Justify your answer. You may use the blank columns in the table above, if needed for your analysis.

1 point	Quantities that needed to be compared were computed (merely computing irrelevant quantities does not count)
1 point	Made clear that analytic procedure described in part (b) was carried out
1 point	Logically concluded that the data were consistent with Alice's claim

$m_1 v_{1,i} + m_2 v_{2,i}$ and $m_1 v_{1,f} + m_2 v_{2,f}$ in the rightmost two columns of the table were computed. For example, for trial 1,

$$m_1 v_{1,i} + m_2 v_{2,i} = (0.30)(0.21) + (0.53)(0.00) = 0.063\ \text{kg} \cdot \frac{\text{m}}{\text{s}}$$

The differences between $m_1 v_{1,i} + m_2 v_{2,i}$ and $m_1 v_{1,f} + m_2 v_{2,f}$ are of the order of $0.005\ \text{kg} \cdot \frac{\text{m}}{\text{s}}$.

The uncertainty ($\pm 0.02\,\frac{\text{m}}{\text{s}}$) in the velocity means that repeated trials might yield similar data with velocities off by about $\pm 0.02\,\frac{\text{m}}{\text{s}}$. For example, a repeated trial might yield data similar to data from trial 1, perhaps with slightly different value of $0.23\,\frac{\text{m}}{\text{s}}$, rather than $0.21\,\frac{\text{m}}{\text{s}}$, for $v_{1,i}$. The quantity $m_1 v_{1,i} + m_2 v_{2,i}$ would be

$$\left(m_1 v_{1,i} + m_2 v_{2,i}\right)_{\text{PLAUSIBLE}} = (0.30)(0.23) + (0.53)(0.00) = 0.069\ \text{kg} \cdot \frac{\text{m}}{\text{s}}$$

so the $m_1 v_{1,i} + m_2 v_{2,i}$ in trial 1 has an uncertainty of roughly

$$\left|\left(m_1 v_{1,i} + m_2 v_{2,i}\right)_{\text{PLAUSIBLE}} - \left(m_1 v_{1,i} + m_2 v_{2,i}\right)_{\text{OBS}}\right| = \left|0.069\ \text{kg} \cdot \frac{\text{m}}{\text{s}} - 0.063\ \text{kg} \cdot \frac{\text{m}}{\text{s}}\right|$$
$$= 0.006\ \text{kg} \cdot \frac{\text{m}}{\text{s}}$$

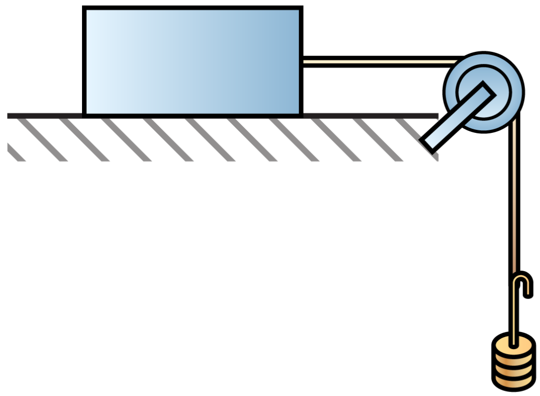
Taking this estimated uncertainty for trial 1 as an order-of-magnitude estimate of the uncertainties for all the $m_1 v_1 + m_2 v_2$ for all the trials leads to the conclusion that in each trial, the total pre- and post-collision $m_1 v_1 + m_2 v_2$ are equal (roughly within experimental uncertainty).

We can identify a magnitude P_C, for example $P_C = 0.25\ \text{kg} \cdot \frac{\text{m}}{\text{s}}$, so that every row (all of them) in the table with $\left|m_1 v_{1,i} + m_2 v_{2,i}\right| < P_C$ has a value of $m_1 v_{1,f} + m_2 v_{2,f}$ equal to $m_1 v_{1,i} + m_2 v_{2,i}$ (the two sums differ by roughly no more than the uncertainty in $m_1 v_1 + m_2 v_2$) and every row (there are none) in the table with a magnitude of $\left|m_1 v_{1,i} + m_2 v_{2,i}\right| \geq P_C$ has a value of $m_1 v_{1,f} + m_2 v_{2,f}$ with the same sign as $m_1 v_{1,i} + m_2 v_{2,i}$ and with $\left|m_1 v_{1,f} + m_2 v_{2,f}\right| = P_C$. **Thus, the data are consistent with Alice's claim.**

Expected wrong answer and comment: It is expected that some students will mistakenly conclude that the data are inconsistent with Alice's claim because the students noticed that absent was a bunch of rows with a variety of particularly high post-collision momentum magnitudes that had total post-collision momenta with a single common magnitude that could have been identified as P_C.

1.4.4 **PLR.** Correcting false reasoning: Impulse-momentum

A group of students is shown a block, which is connected to a taut massless string that is draped around a massless and frictionless pulley and which is connected to a slotted mass set, as shown in the figure below. The block is on a horizontal, frictionless tabletop.



Initially, an instructor holds the block in place. The students are asked to predict how the horizontal momentum of the system consisting of the block, string, and slotted mass set will (or won't) change after the instructor releases the block. A group of students complains that there is an apparent contradiction:

> *Without the gravitational force that the Earth exerts on the slotted mass set, the system would remain at rest, so the gravitational force that the Earth exerts on the slotted mass set is the cause of the motion of the system. The gravitational force that the Earth exerts on the slotted mass set is directed purely in the vertically downward direction, so the horizontal component of the impulse on the system is zero. Thus, the total horizontal momentum of the system should stay constant. However, the block moves faster and faster toward the right (while the slotted mass set moves faster and faster toward the bottom of the page) after the block is released, so the total horizontal momentum of the system actually increases in magnitude.*

In a clear, coherent paragraph-length response that refers to the horizontal component of the impulse on the system, explain why the total horizontal momentum of the system changes after the block is released even though the gravitational force that the Earth exerts on the slotted mass set is purely vertical.

1.5 Work-energy

1.5.1 *ExptDes.* Design so simple, easy to forget to mention principle: KE of cart

The following data were obtained for a cart sliding across a horizontal table. The table coincides with the xy plane.

Mass of cart (kg)	x-component of velocity (m/s)	y-component of velocity (m/s)
0.33 kg	0.30 m/s	0.40 m/s

(a) Describe an analytic procedure for using these data to obtain the kinetic energy of the cart. Give enough detail so that another student who was absent from school on the day kinetic energy was learned can carry out your procedure.

(b) Explain how you know that the method you described in part (a) will give the kinetic energy of the cart.

1.5.2 **PLR.** Work-kinetic energy.

A block of mass m is initially at rest on a horizontal, frictionless track. The block is pushed toward the right by a force of constant strength F through a distance $\Delta\ell$, immediately after which the speed of the block is v_1. In a second experiment, a block of mass $3m$ is initially at rest on the same track and then pushed toward the right by a force, again of constant strength F, again through a distance $\Delta\ell$, immediately after which the speed of the block of mass $3m$ is v_2. Indicate how the speeds v_1 and v_2 compare:

___ $v_1 > v_2$ ___ $v_1 < v_2$ ___ $v_1 = v_2$

In a clear, coherent paragraph-length response, explain your choice in terms of energy principles.

1.5.2.1 Key

A block of mass m is initially at rest on a horizontal, frictionless track. The block is pushed toward the right by a force of constant strength F through a distance $\Delta\ell$, immediately after which the speed of the block is v_1. In a second experiment, a block of mass $3m$ is initially at rest on the same track and then pushed toward the right by a force, again of constant strength F, again through a distance $\Delta\ell$, immediately after which the speed of the block of mass $3m$ is v_2. Indicate how the speeds v_1 and v_2 compare:

✔ $v_1 > v_2$ ___ $v_1 < v_2$ ___ $v_1 = v_2$

In a clear, coherent paragraph-length response, explain your choice in terms of energy principles.

[There is a way to answer this question without using energy principles (e.g. Newton's 2nd law), but the problem statement explicitly requested a response in terms of energy principles].

1 point	Indicated energy conservation principle (at least in name). **Note:** Merely stating $F\Delta\ell = \frac{1}{2}mv^2$ does not satisfy this requirement because $F\Delta\ell = \frac{1}{2}mv^2$ can be obtained from the timeless kinematics equation without any knowledge of energy principles.
1 point	Indicated definition of kinetic energy $K = \frac{1}{2}mv^2$ Merely having $\frac{1}{2}mv^2$ appear "somewhere" in the paragraph response does not, by itself, show that the student knew that $\frac{1}{2}mv^2$ was equal to (by definition) kinetic energy.
1 point	Used energy conservation principle, kinetic energy definition, and definition of work to conclude that $\Delta W = K_f = \frac{1}{2}mv^2$. (There is an alternative solution in which the student does not declare that the initial kinetic energy K_i is zero, but, rather, merely the same in both experiments. A slightly modified scoring guideline would be used to grade such an alternative solution).
1 point	Indicated definition of work $\Delta W = \sum F_\parallel \Delta\ell$ Merely having $\sum F_\parallel \Delta\ell$ appear "somewhere" in the paragraph response does not, by itself, show that the student knew that $\sum F_\parallel \Delta\ell$ was equal to (by definition) (net) work.
1 point	Used fact that F was same in both experiments and $\Delta\ell$ was same in both experiments to conclude that ΔW was same in both experiments.
1 point	Indicated that K_f was same in both experiments and so, because the mass of the block increased, the final speed of the block had to decrease.
1 point	(Trash can point): For a response that follows the published guidelines for paragraph-length responses for AP Physics 1 and 2.

The system is the block. According to energy conservation, the initial total mechanical energy of the system plus the work done equals the final total mechanical energy (after block has just traveled through distance $\Delta\ell$) plus the change ΔU_{INT} in the internal energy. There was no frictional heating of any object, so the $\Delta U_{\text{INT}} = 0$. Kinetic energy K is defined as $K = \frac{1}{2}mv^2$, so the initial kinetic energy (when block was starting out at rest, $v = 0$) is $K_i = \frac{1}{2}m \cdot 0^2 = 0$. Thus, the work done ΔW becomes fully converted into the final (just after block has traveled through distance $\Delta\ell$) kinetic energy K_f.

The work on the block is defined as $\Delta W = \sum F_\parallel \Delta\ell$, where $\sum F_\parallel$ stands for the scalar component of net force on the block along the direction of the block's displacement (here, horizontal). The horizontal component of the net force on the block is equal to the applied force of strength F. The $\Delta\ell$ in $\Delta W = \sum F_\parallel \Delta\ell$ is the distance through which the force is applied, which is actually labeled $\Delta\ell$ in the problem statement. Because neither F nor $\Delta\ell$ were changed in the second experiment, the work done ΔW in the second experiment equaled the work done in the first experiment. Because ΔW was the same in both experiments, the final kinetic energy K_f, which equals $\frac{1}{2}mv_1^2$ in experiment 1 and $\frac{1}{2}(3m)v_2^2$ in experiment 2, was the same in both experiments. The mass m was increased (to become $3m$). So, the speed must have decreased ($v_1 > v_2$).

1.5.3 **PLR.** Simple explanation: Roller coaster

A cart of mass m moves along a frictionless roller coaster track. At one position, the height of the cart above the ground is H, and at another position, the height of the cart is $h < H$. Indicate how the speeds v_h and v_H of the cart, at those respective positions, compare.

___ $v_h > v_H$ ___ $v_h < v_H$ ___ $v_h = v_H$

In a clear, coherent paragraph-length response, explain your choice.

1.5.3.1 Key

A cart of mass m moves along a frictionless roller coaster track. At one position, the height of the cart above the ground is H, and at another position, the height of the cart is $h < H$. Indicate how the speeds v_h and v_H of the cart, at those respective positions, compare.

✔ $v_h > v_H$ ___ $v_h < v_H$ ___ $v_h = v_H$

In a clear, coherent paragraph-length response, explain your choice.

1 point	Indication of energy conservation principle (at least by name). **Note:** Implicit use by writing $\frac{1}{2}mv_H^2 + mgH = \frac{1}{2}mv_h^2 + mgh$ does not alone satisfy this requirement because the equation $\frac{1}{2}mv_H^2 + mgH = \frac{1}{2}mv_h^2 + mgh$ can be obtained by a naïve misapplication of the timeless equation from kinematics.
1 point	Used energy conservation principle to demonstrate $K_H + U_{G,H} = K_h + U_{G,h}$.
1 point	Explicitly cited and used $U_G = mgh$ to conclude that since mass m is constant and $H > h$, $U_{G,H} > U_{G,h}$.
1 point	Used $U_{G,H} > U_{G,h}$ and $K_H + U_{G,H} = K_h + U_{G,h}$ to conclude that $K_H < K_h$.
1 point	Explicitly cited and used $K = \frac{1}{2}mv^2$ to conclude that since mass m is constant and $K_H < K_h$, $v_H < v_h$.

The system is the cart and the Earth. According to energy conservation, the total mechanical energy of the system at height H plus the work done on the system going from the snapshot at height H to the snapshot at height h equals the total mechanical energy of the system at height h plus the change in internal energy going from the snapshot at H to the snapshot at h. The roller coaster track is frictionless, so the work is zero. No thermal energy is being developed in the system, so $\Delta U_{\text{INT},H\to h} = 0$. Thus, the total mechanical energy at H, $K_H + U_{G,H}$ equals the total mechanical energy at h, $K_h + U_{G,h}$.

Gravitational potential energy of an object and the Earth is $U_{G,E\&O} = mgh$, so at heights H and h, respectively, the gravitational potential energies are $U_{G,H} = mgH$ and $U_{G,h} = mgh$. Because the mass of the cart m is constant and because $H > h$, the gravitational potential energy is greater at height H: $U_{G,H} = mgH > mgh = U_{G,h}$.

So, in order for total mechanical energies at heights H and h to be equal, it must be that the kinetic energy is less at height H: $K_H < K_h$.

Kinetic energy K is defined as $K = \frac{1}{2}mv^2$, so at heights H and h, respectively, the kinetic energies of the cart are $K_H = \frac{1}{2}mv_H^2$ and $K_h = \frac{1}{2}mv_h^2$. The mass of the cart m is constant, so the way in which the kinetic energy at H is less must be that $v_H < v_h$.

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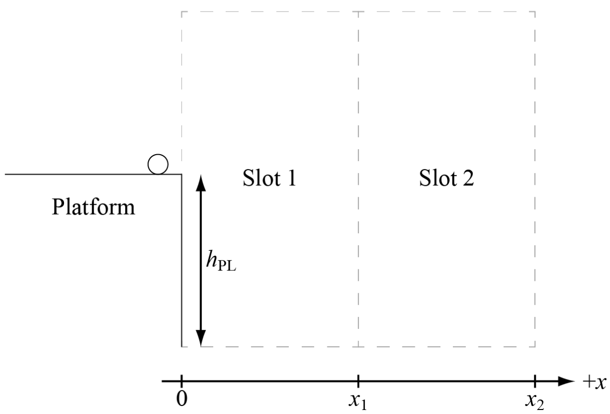
1.5.4 (25 min) **QQT.** Toys in slots.

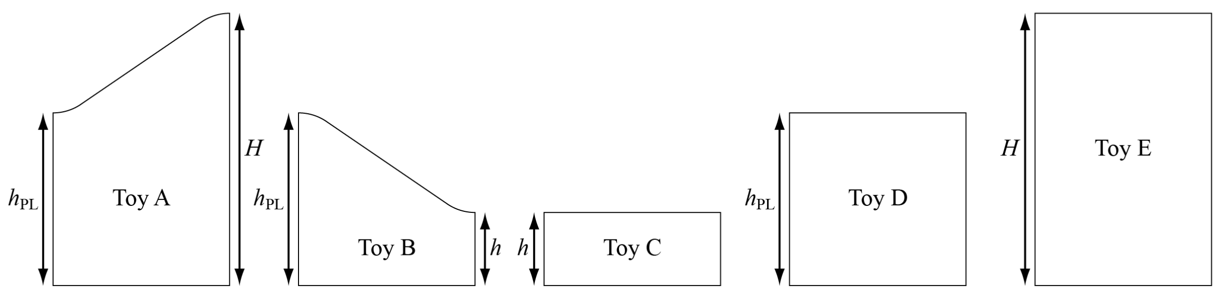
A small frictionless ball slides with speed v_0 across a horizontal platform at height h_{PL} above the ground toward two regions of equal horizontal width labeled Slot 1 and Slot 2. A group of children is offered one copy each of toys A-E, which they can insert into the slots. The children are not allowed to rotate the toys. The children are instructed to insert one toy into each slot so as to satisfy two conditions.



Condition 1: The speed of the ball just as the ball passes horizontal position $x = x_2$ must have the smallest possible value.

Condition 2: From among those toy selections that satisfy condition 1, the children must then choose the assignment of toys to slots that minimizes the time the ball takes to travel from horizontal position $x = 0$ to horizontal position $x = x_2$.

At all times until passing horizontal position $x = x_2$, the ball must be in contact with the platform or a toy.



(a)

i. Based on condition 1 alone, which toy is guaranteed to be inserted into a slot? (Check one option).

___ A ___ B ___ C ___ D ___ E

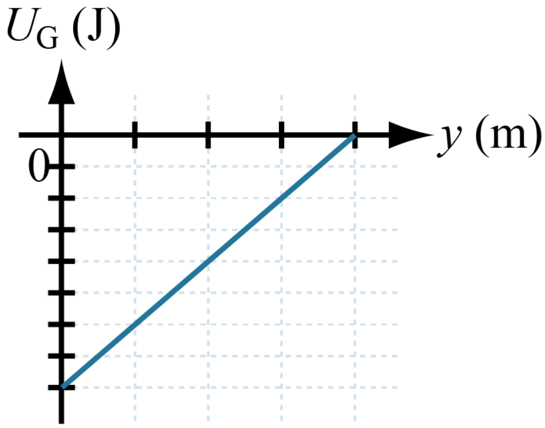
Explain your reasoning.

ii. Now considering also condition 2, which toys should the children place in the slots?

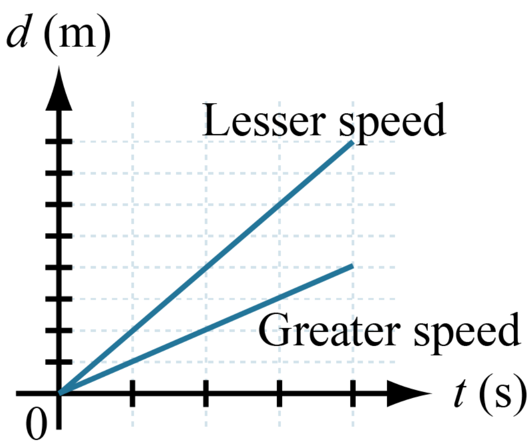
Slot 1 should contain toy ____ Slot 2 should contain toy ____

Explain your reasoning.

(b) On the Internet, the children find the following graph for the gravitational potential energy of an object-Earth system as a function of object height y above the Earth. Regardless of whether the graph is actually correct, is the graph consistent with your reasoning in part (a)? Explain.

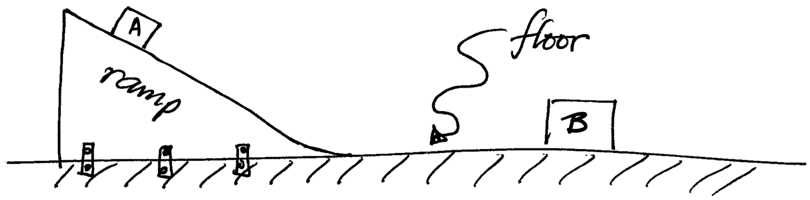


(c) On the Internet, the children also find the following graph relating the distance traveled to the elapsed travel time and the speed at which travel proceeds. Regardless of whether the graph is actually correct, is the graph consistent with your reasoning in part (a)? Explain.



(d) One of the children attempts to compute the x-velocity $v_{x,1}$ that the ball would have at $x = x_1$ if Toy B were placed in Slot 1. The child argues that the speed at $x = x_1$ can be computed using the timeless equation $v_{x,i}^2 + 2a_x\Delta x = v_{x,f}^2$ by letting the initial x-velocity be v_0, the final x-velocity be $v_{x,1}$, recognizing that the acceleration is $-g$, and recognizing that the displacement is $\Delta x = h - h_{\text{PL}}$. Identify a feature of the child's reasoning, if any, that is incorrect. If the child's reasoning is correct, say so. Justify your answer.

1.5.5 **PLR.** Tempt student to misapply conservation principle: Block, incline, block
Block A of mass m_{A} is released from a height H on a frictionless inclined ramp that is attached to the horizontal, frictionless floor, upon which a block B of mass m_{B} initially rests.



(a) At the moment that the block A is released, how do the gravitational potential energy $U_{\mathrm{G,A\&E}}$ of the block A-Earth system and the gravitational potential energy $U_{\mathrm{G,B\&E}}$ of the block B-Earth system compare?

____ $U_{\mathrm{G,A\&E}} > U_{\mathrm{G,B\&E}}$ ____ $U_{\mathrm{G,A\&E}} < U_{\mathrm{G,B\&E}}$ ____ $U_{\mathrm{G,A\&E}} = U_{\mathrm{G,B\&E}}$

Briefly explain your reasoning.

The foot of the ramp is smoothly curved (so that block A smoothly transfers onto, rather than slams into, the floor). After leaving the ramp, block A collides with and sticks to block B. The resulting speed of the stuck-together blocks $v_{\mathrm{A\&B}}$ is measured. The experiment is repeated, but now with block A having a greater mass m_{A}.

(b) Indicate whether the kinetic energy K_2 of the stuck-together blocks in the new experiment is greater than, less than, or equal to the kinetic energy K_1 of the stuck-together blocks in the original experiment.

____ $K_2 > K_1$ ____ $K_2 < K_1$ ____ $K_2 = K_1$

Explain your reasoning.

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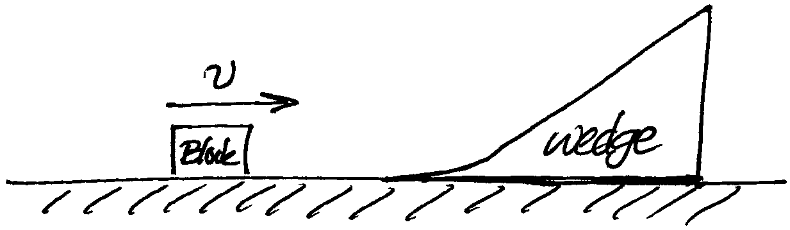
1.5.6 **PLR.** Block at unanchored wedge

As shown in the figure below, a block of mass m slides across a horizontal frictionless surface with speed v toward a wedge of mass $M > m$ that is initially at rest on the horizontal surface.

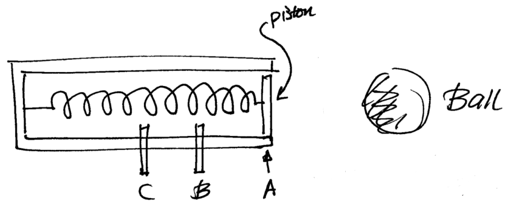


The inclined top surface of the wedge is frictionless. The block slides up the wedge and eventually reaches a maximum height H. The experiment is repeated, but now with the mass of the block equal to M and the mass of the wedge equal to m. Will the new maximum height of the block on the wedge be greater than, less than, or equal to H from the first experiment?

__ Greater than __ Less than _ Equal

Explain your reasoning.

1.5.7 **PLR.** Tempt student to misassume system: Piston, spring, ball launcher
A horizontally-oriented launcher (below) consists of a massless spring and an attached thin, massless piston housed in a capped frictionless tube. When the piston is at position A, the spring is relaxed. The spring can be compressed by using thin pins to hold the piston at positions B and C.



(a) In one experiment, a ball is shoved into the launcher, and a pin is used to keep the spring compressed with the ball at rest, touching the piston, with the piston staying at position B. In a second experiment, the same procedure is used, except with the ball resting with the piston at position C, not B. Indicate how the mechanical energy $E_{\text{MECH,B}}$ of the ball at the end of the first experiment compares with the mechanical energy $E_{\text{MECH,C}}$ of the ball at the end of the second experiment.

__ $E_{\text{MECH,C}} > E_{\text{MECH,B}}$ __ $E_{\text{MECH,C}} < E_{\text{MECH,B}}$ __ $E_{\text{MECH,C}} = E_{\text{MECH,B}}$

Briefly explain your reasoning.

The launcher is now used to launch the ball. First, the ball is horizontally launched from rest with the piston released from position B. The ball exits the launcher with speed $v_{\text{B,HOR}}$ as the piston passes through position A. In a second trial, the ball is still launched from rest with the piston released from position B, but now with the launcher oriented vertically (to launch the ball upward). The ball again exits the launcher, now with speed $v_{\text{B,VER}}$ as the piston passes through position A.

(b) Indicate how the muzzle speeds $v_{\text{B,HOR}}$ and $v_{\text{B,VER}}$ compare.

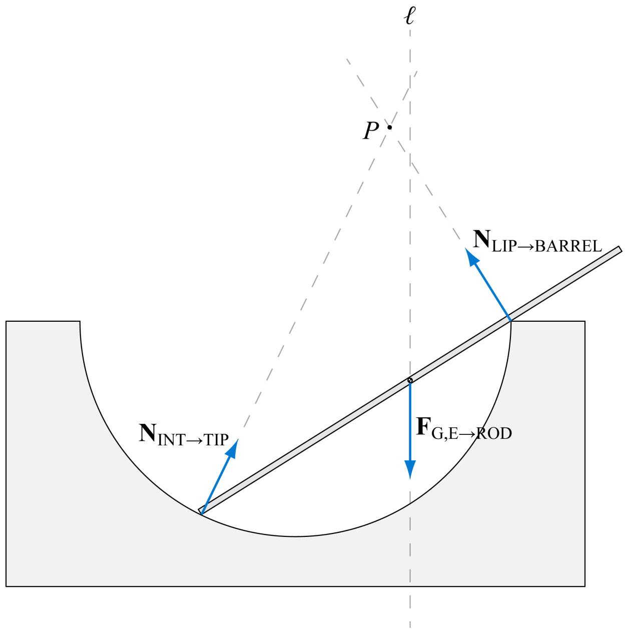
__ $v_{\text{B,HOR}} > v_{\text{B,VER}}$ __ $v_{\text{B,HOR}} < v_{\text{B,VER}}$ __ $v_{\text{B,HOR}} = v_{\text{B,VER}}$

Explain your reasoning.

1.6 Rotational statics

1.6.1 **PLR.** Freedom to choose unusual axis of rotation: Adaptation of classic rod-in-bowl

The figure below shows a cutaway view of a frictionless hemispherical bowl and a rod. The tips of the rod and the lip of the bowl are rounded and *slightly* elastic. As a result, the normal force $\vec{\mathbf{N}}_{\text{INT}\to\text{TIP}}$ that the interior of the bowl exerts on a tip of the rod is perpendicular to the interior of the bowl, and the normal force $\vec{\mathbf{N}}_{\text{LIP}\to\text{BARREL}}$ that the lip of the bowl exerts on the barrel of the rod is perpendicular to the rod. Lines drawn through these two normal force vectors intersect at point P. Vertical line ℓ is drawn through the gravitational force vector $\vec{\mathbf{F}}_{\text{G,E}\to\text{ROD}}$ acting from the center-of-mass of the rod. The forces are not drawn to scale, and the figure might not show the rod at static equilibrium.



When the rod is at static equilibrium, will point P lie to the left of line ℓ (as shown), to the right of line ℓ, or on line ℓ? Indicate your choice below.

___ Pt. P is to the left of line ℓ ___ Pt. P is to the right of line ℓ ___ Pt. P is on line ℓ

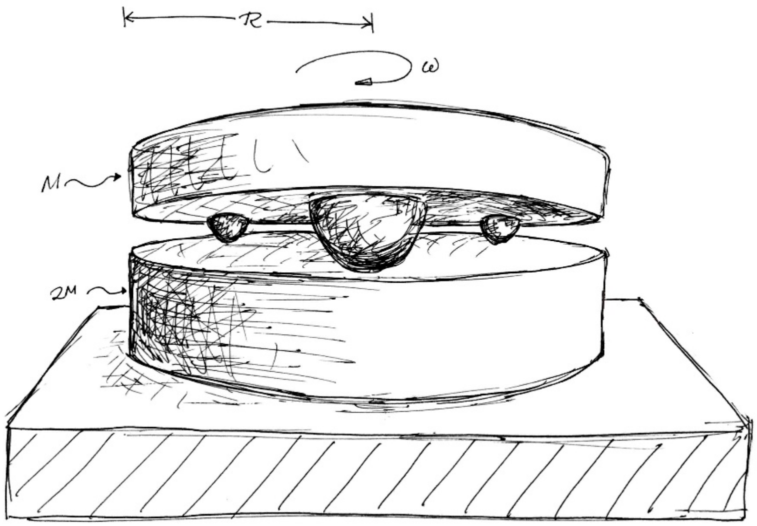
In a clear, coherent paragraph-length response that incorporates physics principles, explain your choice.

1.7 Rotational motion

1.7.1 *Calculation.* Using scaling/graphical reasoning to avoid "I substituted": Vinyl record

A student is listening to a song on a record (a record is a historical technology in which sound waves were mechanically represented using indentations on a vinyl disk) that is on a turntable spinning with angular velocity ω. The student finishes listening to the song and turns the record player off at time $t = 0$. The turntable slows down with constant angular acceleration until it is no longer spinning. Let time $t_{\frac{1}{2}}$ denote the time at which the angular velocity has decreased to ½ its original value, and let t_{STOP} denote the time at which the turntable comes to rest. What is the ratio of the angular displacement of the turntable from time $t_{\frac{1}{2}}$ to time t_{STOP} to the angular displacement of the turntable from time $t = 0$ to time $t_{\frac{1}{2}}$?

1.7.2 **PLR.** Adaptation of linear problem in Liao 2018 TPT article: Stacked disks



The apparatus in the figure above consists of a horizontal surface, a uniform disk of mass $2M$ in contact with the surface, and a second uniform disk of mass M that makes contact with the first disk via three identical massless rubber "feet" equally spaced around the disks' outer rims. Both disks have a radius of R. The rotational inertia of a uniform disk having mass M and radius R is $\frac{1}{2}MR^2$. There is no friction between the horizontal surface and the disk of mass $2M$, but there is friction between the disk of mass $2M$ and the rubber feet of the disk of mass M. Initially, the disk of mass M is spinning with angular speed ω while the underlying disk is stationary. Eventually the two disks are co-spinning with the same angular speed $\omega_{f,1}$.

The experiment is repeated, except now with the masses M and $2M$ both increased by one same factor.

(a) How does the final angular speed $\omega_{f,2}$ achieved by the two disks in the second experiment compare with the final angular speed $\omega_{f,1}$ achieved by the two disks in the first experiment?

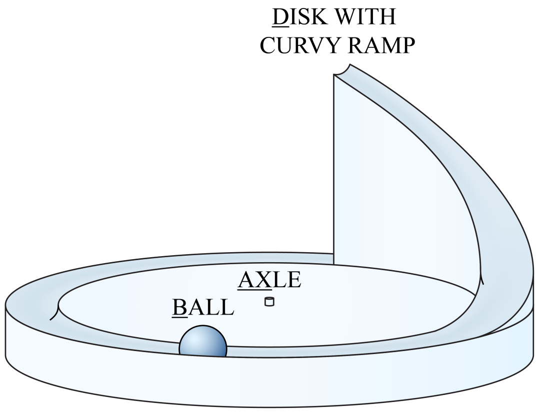
____ $\omega_{f,1} > \omega_{f,2}$ ____ $\omega_{f,1} < \omega_{f,2}$ ____ $\omega_{f,1} = \omega_{f,2}$

Explain your reasoning.

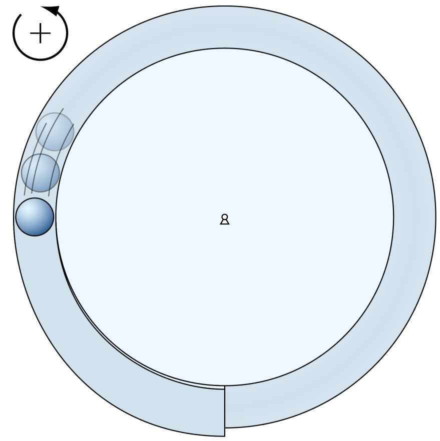
In each experiment, the rubber feet leave skid mark arcs on the lower disk, and the distances between the rubber feet are sufficiently great so that the skid marks do not overlap.

(b) In a clear, coherent, paragraph-length response that incorporates relevant principles of physics, explain whether the skid mark arcs in the second experiment will be longer than, shorter than, or as long as the skid mark arcs in the first experiment.

1.7.3 **PLR.** Lazy Susan.



Perspective side view



Top view

A disk is mounted (figure above, left) horizontally on a frictionless, vertical axle, which serves as the axis of rotation. A thin groove carved out near the outer edge of the disk fits a frictionless ball. A curved ramp is attached to a portion of the outer edge of the disk, and the carved groove smoothly continues into the ramp so that the ball can slide up the ramp. The rotational inertia of the ball with respect to the axle is I_{BALL}, and the rotational inertia of the disk and attached ramp is $I_{\text{DISK}} > I_{\text{BALL}}$. The top view (figure above, right) shows the initial moment of an experiment in which the ball is revolving (but not spinning) counterclockwise with angular momentum of magnitude L_1 with respect to the axle while the disk is not yet rotating. In a second experiment, the ball is initially stationary (and, again, not spinning) while the disk is rotating clockwise with angular momentum of magnitude L_1. The ball is located at the smooth base (not the vertical “cliff”) of the curvy ramp at the beginning of each experiment.

(a) Which of the following two objects has the greatest magnitude of angular velocity?
___ Ball at beginning of first experiment   ___ Disk at beginning of second experiment
Explain your reasoning.

(b) During the first experiment, the ball reaches a maximum height h_1 partway up the ramp. During the second experiment, the ball reaches a maximum height h_2, also partway up the ramp. How do the two maximum heights compare?
___ $h_1 > h_2$ ___ $h_1 < h_2$ ___ $h_1 = h_2$
In a clear, coherent paragraph-length response that incorporates physics principles, explain your reasoning.

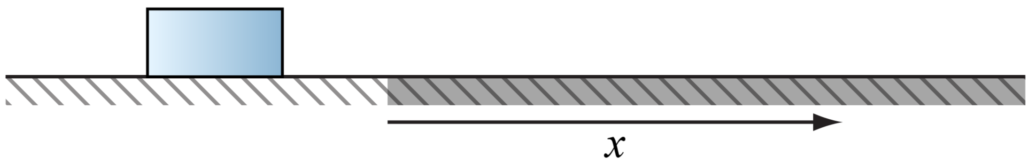
1.7.4 *ExptDes.* Placeholder: Correct false experimental procedure

[Alice claims that by … one can test whether …. Identify an aspect of Alice's procedure that is incorrect. Why is the aspect you identified incorrect? How could you change the aspect of Alice's procedure you identified so that Alice's procedure can be used to test whether …?]

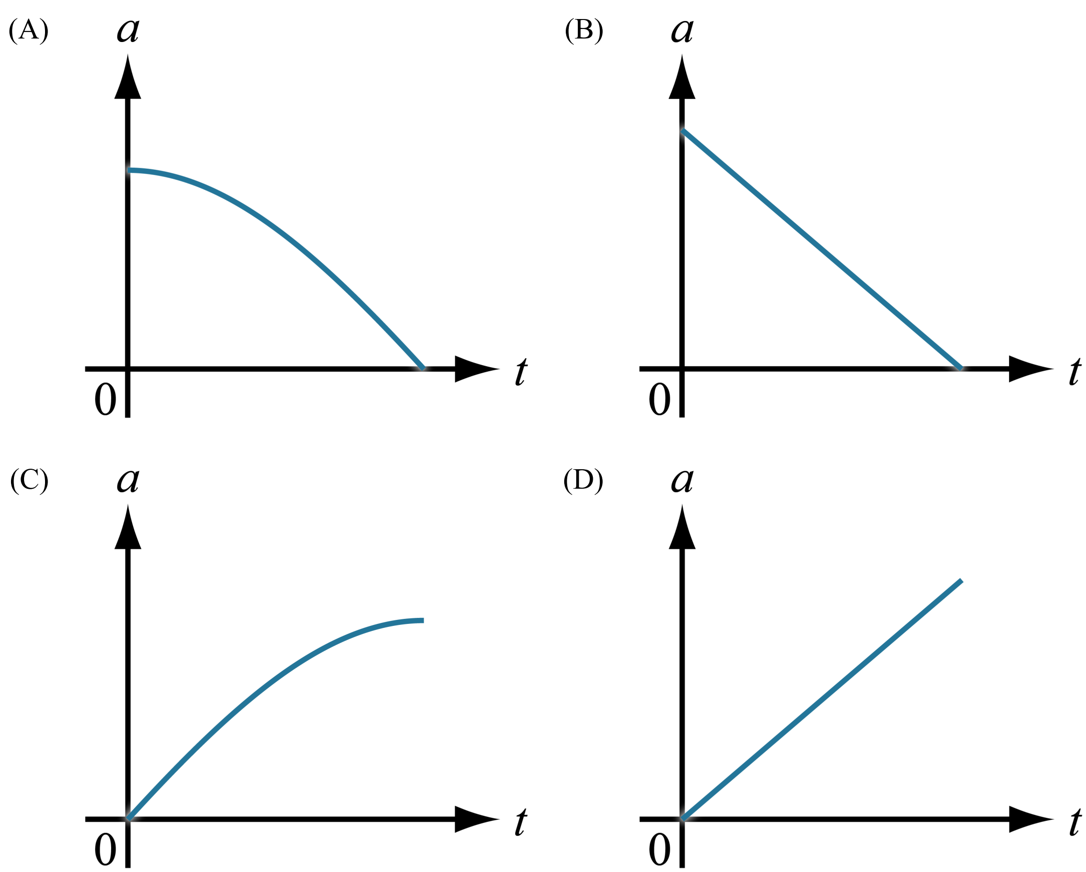
1.8 Simple harmonic motion

1.8.1 **PLR.** Hidden SHM

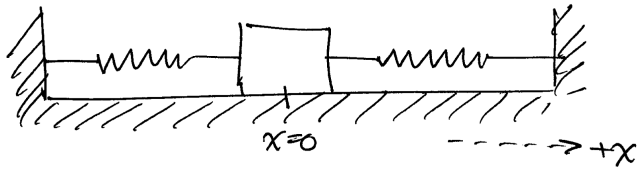
In the figure below, a block is sliding to the right across a frictionless portion of the floor (less shading) and then encounters a second portion of the floor (darker shading). In the darker region, the coefficient of kinetic friction μ_K between the block and the floor increases linearly with distance from the location on the floor where the lighter and darker regions meet according to $\mu_K = bx$, where b is a positive constant with appropriate units. Even though the block is drawn large, consider the block very small (like a point particle), so that at almost any given moment, the block is either entirely on the frictionless surface or entirely on the dark-shaded surface.



Which one of the following graphs represents the magnitude of the block's acceleration as a function of time, beginning at the instant the block first encounters the darker portion of the floor? Explain how you arrived at your choice.



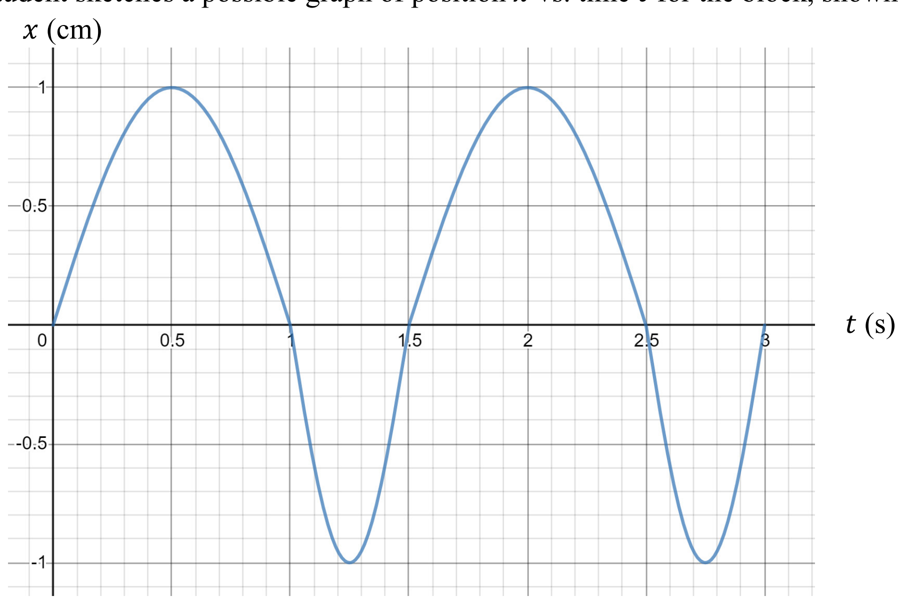
1.8.2 **QQT.** Superficially similar to class question: Spring-mass-spring



In the figure above, a block of mass m is on a frictionless, horizontal floor. The block is in contact with a relaxed massless spring on the left, of spring constant k_L, and in contact with a relaxed massless spring on the right, of spring constant k_R. The other ends of the springs are fixed to immovable walls. The horizontal lengths of the springs can change, but the springs do not bend (the springs remain horizontally oriented). The block is not attached to either spring.

(a) Suppose k_R is much, much greater than k_L. The block is set into oscillatory motion along the x-axis. Derive an algebraic expression for the period of oscillation of the block. Explain how you obtained your result.

(b) Now suppose that k_R is not necessarily much, much greater than k_L, but that k_R and k_L are still not equal. A student sketches a possible graph of position x vs. time t for the block, shown below.

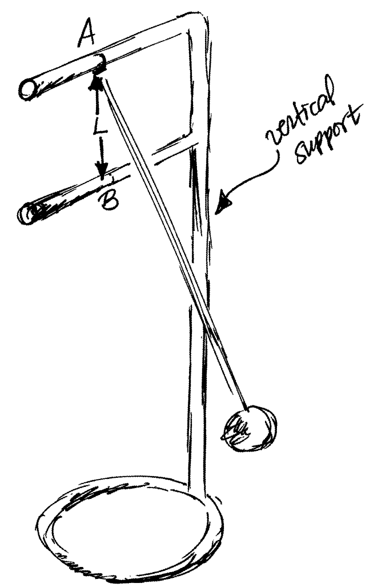


i. Identify two correct geometric features of the student's sketch and explain, using physics principles, why those features are correct.

ii. Identify two incorrect geometric features of the student's sketch and explain, using physics principles, why those features are incorrect.

1.8.3 **PLR.** Unfamiliar scenario: Nasty pendulum thing.

In the apparatus shown to the right, two thin horizontal rods are fixed to an immovable vertical support. One end of a massless string is attached to the upper rod at point A, and the other end of the string is attached to a massive, small "bob" (sphere). The vertical distance between the horizontal rods is L. The figure shows the bob immediately after being released from rest. As the bob swings, the string will come into contact with the lower rod at position B. After passing its lowest position, the bob will then ascend and momentarily come to rest. Assume that the bob is always released from a height lower than the height of position B and that the angle between any slanted segment of the string and the vertical is always "small." The duration of time that elapses between the moment the bob is released from rest and the moment that the bob soonest then again comes to rest is Δt. If L increases, how does Δt change?



__ Increase ___ Decrease _ Stay the same

Explain your reasoning.

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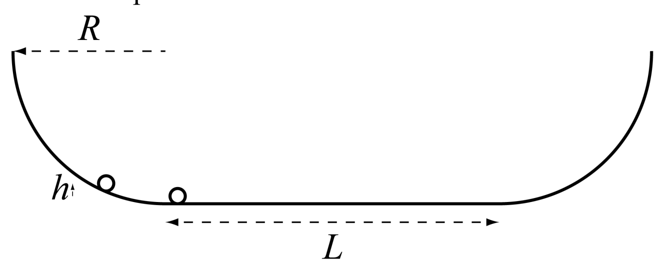
1.8.4 **QQT.** Skateboard stunt ramp



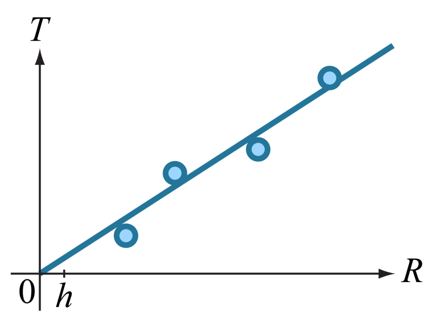
A frictionless skateboarder's stunt ramp consists of a quarter-circle arc having radius of curvature R connected to a horizontal segment of length L connected to another quarter-circle arc having radius of curvature R. A small ball is released from rest on one of the curved portions of the ramp at a height h above the height that the ball has while on the horizontal segment. The figure shows two snapshots at different times of the same ball. The height h is much, much smaller than the radius of curvature R.

(a) The time it takes for the ball to soonest return to the position from which the ball is released is T. Would a slight increase in R result in an increase in T, a decrease in T, or no change in T?

___ Increase in T ____ Decrease in T ___ No change in T

Explain your reasoning.

On the Internet, a student found a scatterplot of T vs. R (below) obtained from a university laboratory experiment.



(b) Regardless of whether the scatterplot is correct, is the scatterplot consistent or inconsistent with your reasoning in your response to part (a) above?

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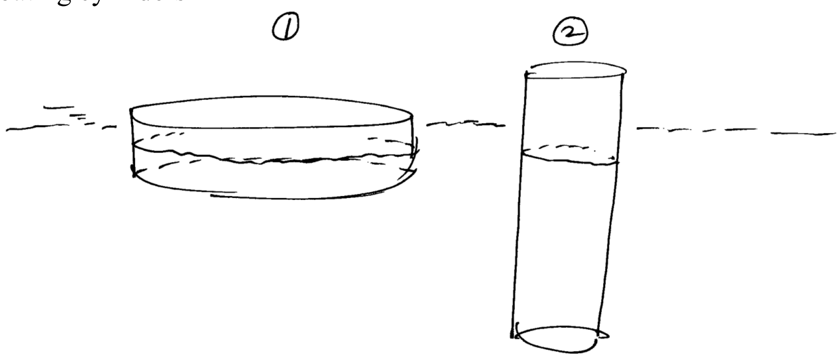
1 AP Physics 2

1.1 Fluid mechanics

1.1.1 **PLR**. Floating cylinders



Two empty cylindrical aluminum cans are floating in water (figure above). Both top caps have been removed. The bottom disk-shaped plate of can 1 is larger than the bottom disk-shaped plate of can 2. An additional mass Δm is deposited into the bottom of can 1, and the same additional mass Δm is also deposited in the bottom of can 2. Care is taken to avoid jostling and overturning the cans. The cans both remain floating, and, after transient oscillations of the cans have damped out, the bottom of can 1 ends up an additional depth d_1 farther below the water surface, and the bottom of can 2 ends up an additional depth d_2 farther below the water surface. How do d_1 and d_2 compare?

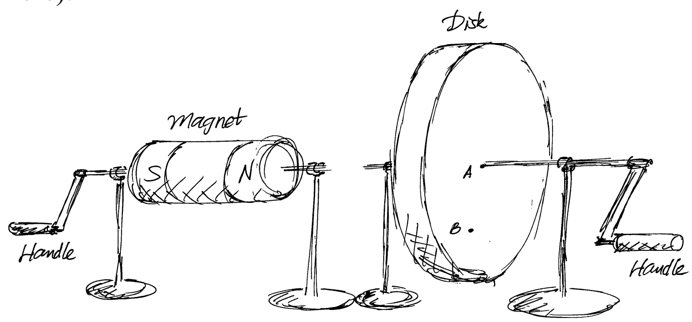
___ $d_1 > d_2$ ___ $d_1 < d_2$ ___ $d_1 = d_2$

In a clear, coherent paragraph-length response that incorporates physics principles, explain your reasoning.

1.2 Electromagnetism

1.2.1 **PLR.** Basically Faraday disk.

(Adapted from physical apparatus my PhD advisor had me play with after seeing scenario described online—I don't know where).



The apparatus above consists of a bar magnet and a copper disk, mounted coaxially. The magnet and, separately, the disk, can be spun using the attached handles. The supporting structures, including the rods threading the magnet and disk, are made of an electrically insulating material. Two dots are inked onto the copper disk, one at position A (near center of disk) and another at position B (farther out from center of disk). For each of the scenarios in the table below, each object is either stationary (not spinning) or in the midst of spinning with constant angular velocity. For each scenario, indicate whether an emf is being sustained between inked positions A and B on the copper disk and explain your reasoning.

Magnet	Disk	Magnitude of emf between inked dots A and B	Explanation of your reasoning (presumably a textbox would be provided) (there is a clever way to write a single paragraph that justifies all four choices)
Still	Still	[ ] Zero [ ] Nonzero	
Still	Spinning	[ ] Zero [ ] Non zero	
Spinning	Still	[ ] Zero [ ] Nonzero	
Spinning	Spinning	[ ] Zero [ ] Non zero	

2 AP PHYSICS C MECHANICS

2.1 Forces/Dynamics

2.1.1 **LFRQ** (FBD, PLR). No need to antidifferentiate: v_y-t graph for object falling with drag.

At initial time $t = 0$, an object of mass m is released from rest. Let the $+y$ direction point vertically downward, with the origin of the y-axis at the location from which the object is released. A drag force from the surrounding fluid has a magnitude $F_{\text{DRAG}}(v)$ that is an increasing function of the speed v with which the object moves relative to the background fluid. The exact form of $F_{\text{DRAG}}(v)$ is not known; in particular, it cannot be assumed that the function $F_{\text{DRAG}}(v)$ is proportional to a power of v.

(a) On the dot below, draw a free-body diagram for the object while the object is falling.

(b) Write, but do not solve, a differential equation that could be numerically solved, were $F_{\text{DRAG}}(v)$ known, to obtain the y-velocity of the object as a function of time. Explain how you used physical principles to arrive at your differential equation.

(c) Without directly integrating the differential equation you wrote down in part (b), explain why the graph of v_y vs. t must be both increasing and concave down for all $t > 0$.

3 AP Physics C Electricity & Magnetism

3.1 Electrostatics

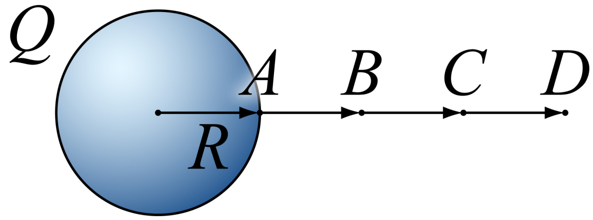
3.1.1 **LFRQ** (PLR). No need to do definite integral: Potential difference outside conducting sphere

A conducting sphere of radius a contains positive charge Q. The electric potential difference $V(a) - V(c)$ between a point outside the sphere at radius $c > a$ and a point on the surface of the sphere is V_0. Suppose a thin spherical sheet of insulating material is constructed at radius $b = \frac{a+c}{2}$ so that the electric field strength at each position between the sphere and the sheet remains unchanged, but so that the electric field strength at each location outside the sheet is now doubled. Will the new potential difference $\left(V(a) - V(c)\right)_f$ in this final situation be greater than, less than, or equal to $\frac{3}{2}V_0$?

___ $\left(V(a) - V(c)\right)_f > \frac{3}{2}V_0$ ___ $\left(V(a) - V(c)\right)_f < \frac{3}{2}V_0$ ___ $\left(V(a) - V(c)\right)_f = \frac{3}{2}V_0$

Explain your reasoning.

3.1.2 **LFRQ** (QQT). Potential difference outside conducting sphere, more like 2017 APP1 FRQ 3.



A conducting sphere of radius R has positive charge Q. Point A is located on the surface of the sphere, point B is located at a distance $2R$ from the center of the sphere, point C is located at a distance $3R$ from the center of the sphere, and point D is located at a distance $4R$ from the center of the sphere, with the center of the sphere and points A, B, C, and D all lying on a single line. The electric potential V_A at position A is greater than the electric potential V_D at position D by an amount $\Delta V_{D\to A}$. Region I is between points A and B, region II is between points B and C, and region III is between points C and D.

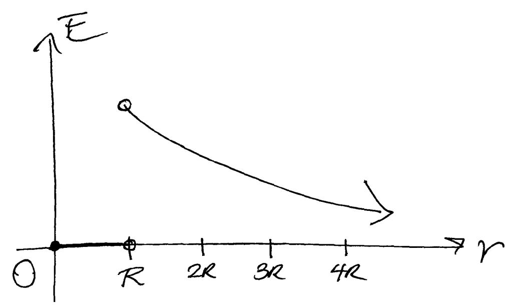
(a) Suppose it could be arranged for the electric field strengths only in region I to be doubled, or, instead, for the electric field strengths only in region II to be doubled, or, instead, for the electric field strengths only in region III to be doubled. In which one of the three regions should the electric field strengths be doubled to increase the potential difference $\Delta V_{D\to A}$ between positions D and A by the greatest amount?

___ Region I only ___ Region II only ___ Region III only

Explain your reasoning

(The apparatus is kept in its original configuration, meaning that none of the modifications offered as choices in part (a) have been applied).

On the Internet, a student found the following sketch of the strength of the electric field as a function of distance r from the center of a conducting sphere.

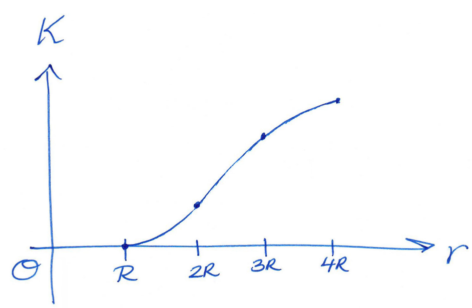


(b) Regardless of whether the plot is actually correct, is the plot from the Internet consistent with your reasoning in part (a) above?

___ Consistent ___ Inconsistent

Explain your reasoning.

On the Internet, a student found the following sketch of the kinetic energy K of a positively-charged particle released from rest just outside of the conducting sphere.



(c) Regardless of whether the plot is actually correct, is the plot from the Internet consistent with your reasoning in part (a) above?

___ Consistent ___ Inconsistent

Explain your reasoning.

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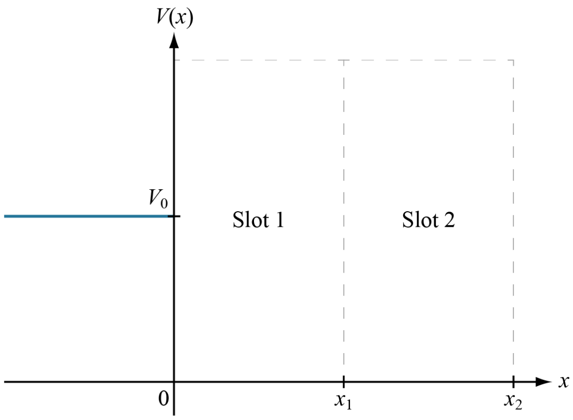
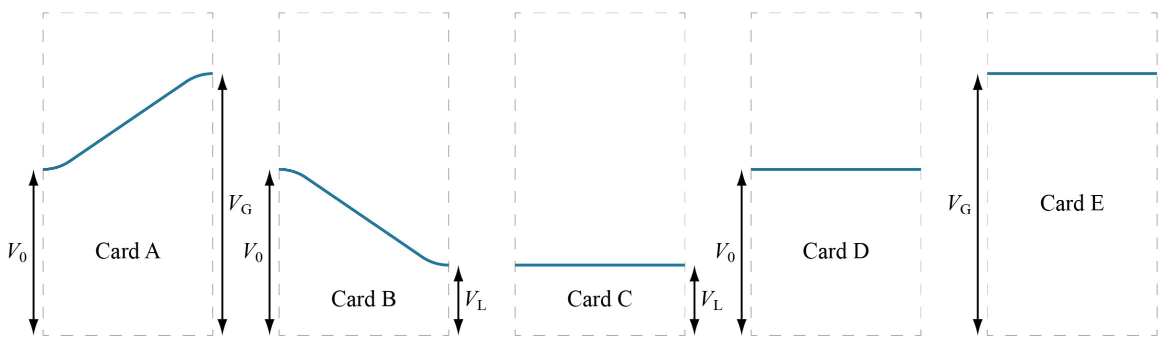
3.1.3 (25 min) **QQT.** Drawing a potential function. (includes optional mechanics part)
A group of children is given an incomplete graph of electric potential $V(x)$ as a function of horizontal position x. A proton, acted on by no force except the electrostatic force associated with $V(x)$, glides horizontally with speed v_0 through the region $x \leq 0$, where $V(x)$ has constant positive value V_0 toward two regions of equal horizontal width labeled Slot 1 and Slot 2. The group of children is offered one copy each of cards A-E, which they can insert into the slots. Each card represents a portion of a V vs. x plot. The children are not allowed to rotate the cards. Assume that the proton can reach horizontal position $x = x_2$ for any assignment of cards to slots. The children are instructed to insert cards into the slots to complete a graph of a potential function $V(x)$ satisfying three conditions.



Condition 1: $V(x)$ is continuous.

Condition 2: The speed of the proton just as the proton passes horizontal position $x = x_2$ must have the smallest possible value.

Condition 3: From among those card selections that satisfy conditions 1 and 2, the children must then choose the assignment of cards to slots that minimizes the time the proton takes to travel from horizontal position $x = 0$ to horizontal position $x = x_2$.

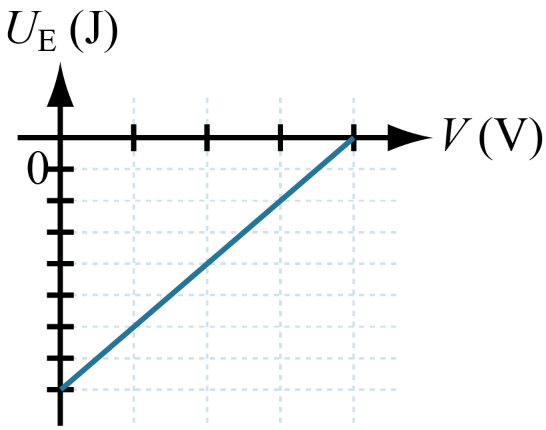


(a)

i. Based on conditions 1 and 2 only, which one card is guaranteed to be inserted into a slot?
___ A ___ B ___ C ___ D ___ E
Explain your reasoning.

ii. [This part can be omitted if you are taking only EM (not Mechanics) this year] Now considering also condition 3, which cards should the children place in the slots?
Slot 1 should contain card ____ Slot 2 should contain card ____
Explain your reasoning.

(b) On the Internet, the children find the following graph for the electric potential energy of a positive-object-neighboring-charges system as a function of electric potential V at the object's location. Regardless of whether the graph is actually correct, is the graph consistent with your reasoning in part (a)? Explain.

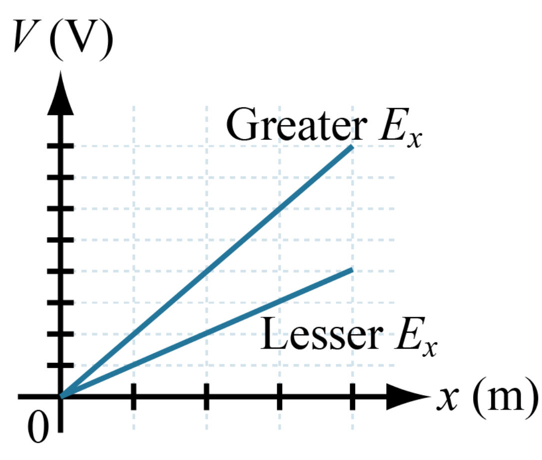


(c) An instructor wants the children to practice designing laboratory setups that could be used to create different V vs. x profiles. The children are asked to assume that charge, if any, is distributed in three-dimensional space such that charge density $\rho(x)$ is a function of horizontal position x alone. One of the students makes the following claim:

Card C shows a portion of a V vs. x graph in which V is constant and positive, so the charge density $\rho(x)$ should also be constant and positive in the region described by Card C.

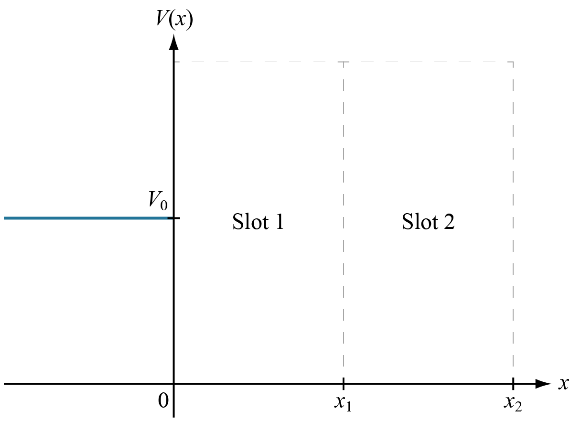
Challenge the student by starting with an assumption of a constant, positive charge density $\rho(x)$ in the region described by Card C.

(d) On the internet, the children find a figure showing graphs of V vs. x for two scenarios. In each scenario, the x-component of the electric field E_x is constant, but the constant E_x values from the two scenarios are not the same. Regardless of whether the figure is actually correct, is the figure consistent with your reasoning in part (c)? Explain.



End-of-Year (EoY) FRQ practice 2021/05/07 21h37 EDT

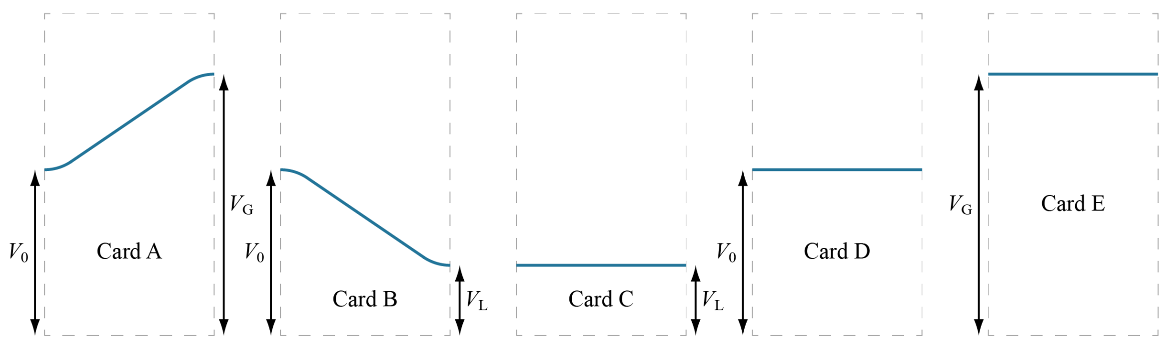
3.1.4 (25 min) **QQT.** Drawing a potential function. (version without mechanics part)
A group of children is given an incomplete graph of electric potential $V(x)$ as a function of horizontal position x. A proton, acted on by no force except the electrostatic force associated with $V(x)$, glides horizontally with speed v_0 through the region $x \leq 0$, where $V(x)$ has constant positive value V_0 toward two regions of equal horizontal width labeled Slot 1 and Slot 2. The group of children is offered one copy each of cards A-E, which they can insert into the slots. Each card represents a portion of a V vs. x plot.



The children are not allowed to rotate the cards. Assume that the proton can reach horizontal position $x = x_2$ for any assignment of cards to slots. The children are instructed to insert cards into the slots to complete a graph of a potential function $V(x)$ satisfying two conditions.

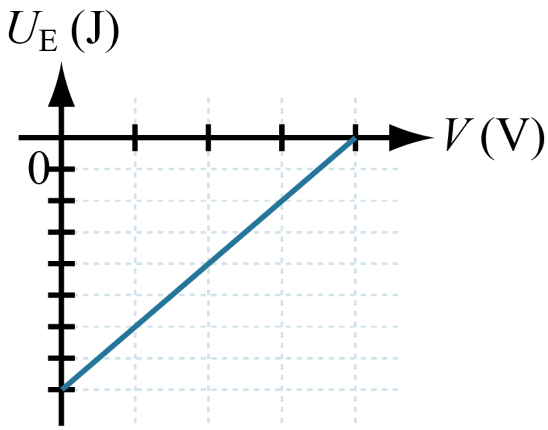
Condition 1: $V(x)$ is continuous.

Condition 2: The speed of the proton just as the proton passes horizontal position $x = x_2$ must have the smallest possible value.



(a) Based on conditions 1 and 2, which one card is guaranteed to be inserted into a slot?
___ A ___ B ___ C ___ D ___ E
Explain your reasoning.

(b) On the Internet, the children find the following graph for the electric potential energy of a positive-object-neighboring-charges system as a function of electric potential V at the object's location. Regardless of whether the graph is actually correct, is the graph consistent with your reasoning in part (a)? Explain.

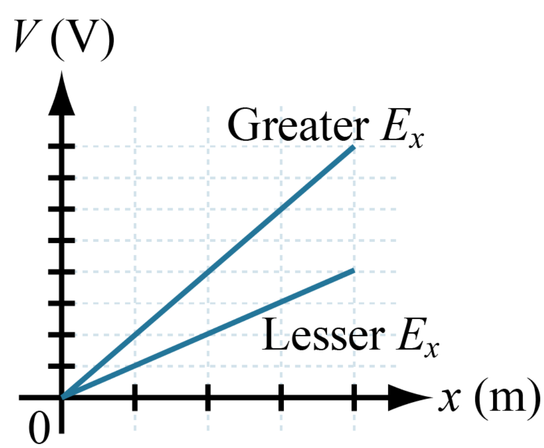


(c) An instructor wants the children to practice designing laboratory setups that could be used to create different V vs. x profiles. The children are asked to assume that charge, if any, is distributed in three-dimensional space such that charge density $\rho(x)$ is a function of horizontal position x alone. One of the students makes the following claim:

> *Card C shows a portion of a V vs. x graph in which V is constant and positive, so the charge density $\rho(x)$ should also be constant and positive in the region described by Card C.*

Challenge the student by starting with an assumption of a constant, positive charge density $\rho(x)$ in the region described by Card C.

(d) On the internet, the children find a figure showing graphs of V vs. x for two scenarios. In each scenario, the x-component of the electric field E_x is constant, but the constant E_x values from the two scenarios are not the same. Regardless of whether the figure is actually correct, is the figure consistent with your reasoning in part (c)? Explain.



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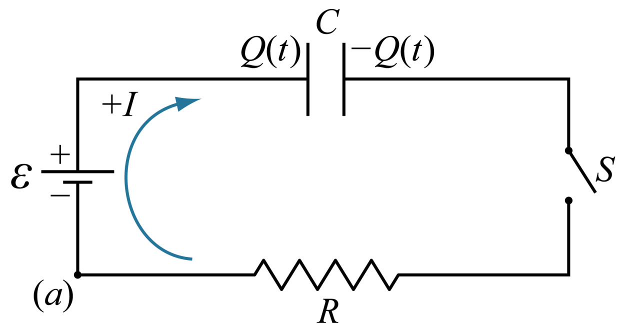
3.2 RC Circuits

3.2.1 **LFRQ** (PLR). No need to antidifferentiate: RC circuit curve sketching

The circuit below consists of a battery of emf $\mathcal{E}$ connected to a capacitor of capacitance C connected to a switch S connected to a resistor of resistance R, which is connected to the battery. The switch is closed at time $t = 0$, at which point the charge on the capacitor is $Q = 0$.



(a) Write, but do not solve, a differential equation that could be solved to find the capacitor charge $Q(t)$ as a function of time. Explain how you used physical principles to arrive at your differential equation.

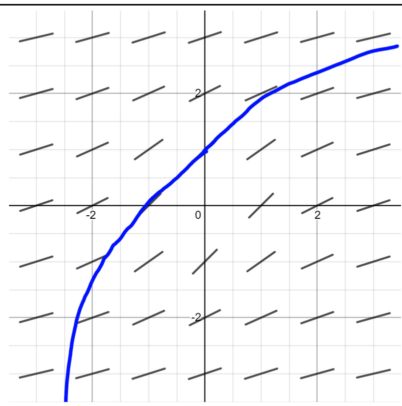
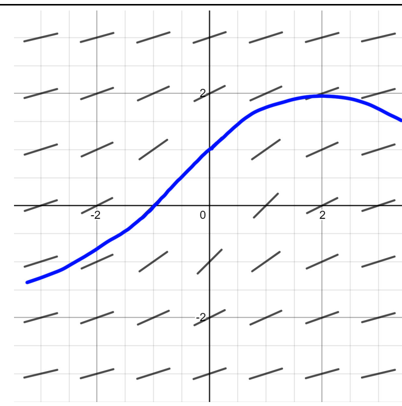
(b) Without referring to the explicit formula for $Q(t)$ that would be obtained by integrating the differentiation equation you wrote in part (a) above, explain why the graph of Q vs. t must be concave down. <u>Merely integrating the differential equation you wrote in part (a) and narrating features of the resultingly derived formula $Q(t)$ will not receive much credit.</u>

4 AP CALCULUS AB

4.1 Differential equations and slope fields

4.1.1 Correcting false reasoning: Solution curve through slope field

Alice and Bob were given a slope field to study for calculus homework. They drew the following particular solutions through (0,1).

Alice's solution curve	Bob's solution curve
	

(a)

i. Indicate two features of each student's solution that are correct and explain why they are correct.

ii. Indicate one feature of each student's solution that is incorrect and explain why it is incorrect.

For the slope field above, the local linearization of the particular solution $y = f(x)$ through (1,2) can be written in the form $y = Ax + B$, where A and B are constants. The local linearization of the particular solution $y = g(x)$ through (1,0) can be written in the form $y = Cx + D$, where C and D are constants.

(b) How do constants A and C compare?
___ $A > C$ ___ $A < C$ ___ $A = C$
Explain your reasoning.

(c) How do constants B and D compare?
___ $B > D$ ___ $B < D$ ___ $B = D$
Explain your reasoning.

Suppose $y = h(x)$ is a particular solution to the differential equation represented by the slope field, but no initial condition is given. Let $j(x) = h(x) + kx$, where k is a constant.

(d) Consider just those points on the slope field that have an illustrated sloped segment. To allow for one or more of these points to be a critical point of $j(x)$, which one of the following conditions should be satisfied?
___ $k > 0$ ___ $k < 0$ ___ $k = 0$ ___ None of the preceding
Explain your reasoning.

(e) Identify a position on the slope field above that could be (or could be near) a point of inflection for a particular solution. Using calculus principles and referring to the slope field, explain your reasoning.

(f) As stated previously, $y = f(x)$ is the particular solution to the slope field through (1,2). Let $H(x) = \int_1^x f(t)\,dt$, for $x > 0$. Is the graph of H vs. x concave up, concave down, or neither at $x = 1$?
___ Concave up ___ Concave down ___ Neither
Explain your reasoning.

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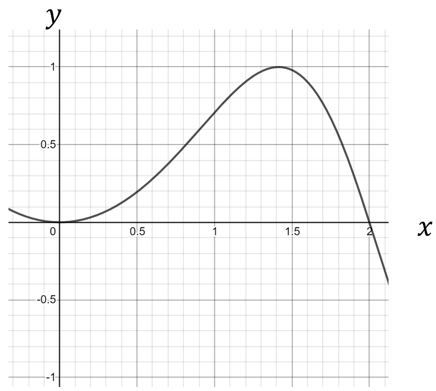
5 AP CALCULUS BC

5.1 Taylor polynomials

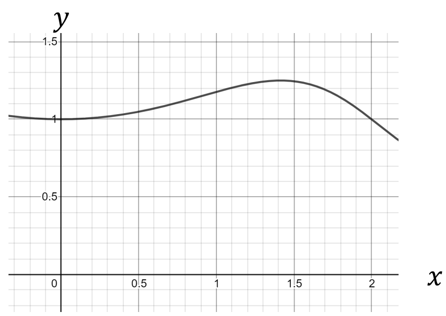
5.1.1 Connecting representations: Graph of f' and Taylor polynomial

Alice shows Bob the graph of f' below and tells Bob the value of $f(1)$. Bob uses the information Alice provides to write down an approximate 2nd-order Taylor polynomial for f centered at $x = 1$.



Alleged graph of f'

Alice realizes she gave Bob the wrong graph and gives Bob the corrected graph of f' below.



Corrected graph of f'

When Bob reconstructs the 2nd-order Taylor polynomial for f centered at $x = 1$, which terms in the Taylor polynomial, if any, change, and why?